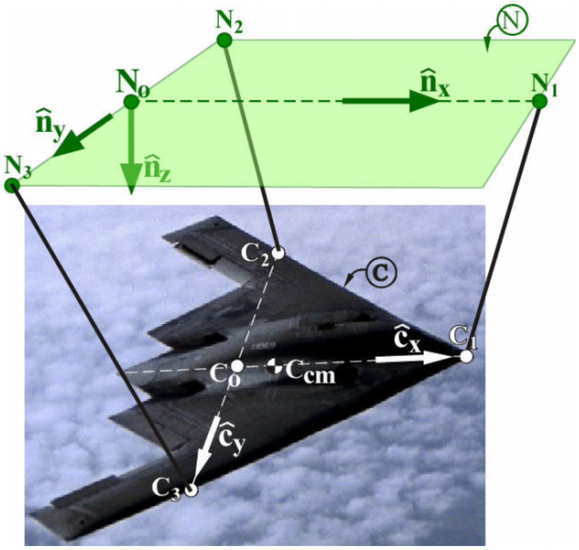
21.9 Statics: Trifilar pendulum for an aircraft.

The following figure shows a rigid aircraft C supported by three thin (massless) inextensible taut cables to a flat horizontal ceiling of an aircraft hanger N (a Newtonian reference frame). The cables attach to N at points N_1, N_2, N_3, and attach to C at points C_1, C_2, C_3, respectively. Point N_o of N is the midpoint of N_2 and N_3. Point C_o of C is the midpoint of C_2 and C_3. Right-handed orthogonal unit vectors $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ and $\hat{\mathbf{c}}_x$, $\hat{\mathbf{c}}_y$, $\hat{\mathbf{c}}_z$ are fixed in N and C, respectively, directed with:

- $\hat{\mathbf{n}}_x$ from N_o to N_1, $\hat{\mathbf{n}}_y$ from N_o to N_3, $\hat{\mathbf{n}}_z$ vertically-downward.
- $\hat{\mathbf{c}}_x$ from C_o to C_1 (aircraft forward direction), $\hat{\mathbf{c}}_y$ from C_o to C_3 (outward along right wing)

Quantity	Symbol	Value
Length of cable $\overline{C_1 N_1}$	L_1	30 m
Length of cable $\overline{C_2 N_2}$	L_2	30 m
Length of cable $\overline{C_3 N_3}$	L_3	30 m
Distance between N_o and N_1	d_N	30 m
Distance between N_o and N_2	w_N	20 m
Distance between C_o and C_1	d_C	30 m
Distance between C_o and C_2	w_C	20 m
Distance between C_o and C_{cm}	d_{cm}	8 m
Mass of aircraft	m	9000 kg
Earth's gravitational acceleration	g	$9.8\ \frac{m}{s^2}$
C's moment of inertia about C_{cm} for $\hat{\mathbf{c}}_x$	I_{xx}	4×10^5 kg m^2
C's moment of inertia about C_{cm} for $\hat{\mathbf{c}}_y$	I_{yy}	3×10^5 kg m^2
C's moment of inertia about C_{cm} for $\hat{\mathbf{c}}_y$	I_{zz}	$I_{xx} + I_{yy}$

Note: C_{cm} is C's center of mass and is along line $\overline{C_o C_1}$.
Note: $\hat{\mathbf{c}}_x$, $\hat{\mathbf{c}}_y$, $\hat{\mathbf{c}}_z$ are parallel to C's principal inertia axes.



- When C is in ***static equilibrium***: Determine the distance between N_o and C_o, tension in each cable, and the aircraft ***"yaw, pitch, and roll angles"*** (described below).

Result when $L_1 = L_2 = 30$ m	distance = 30.0 m	T_1 = 23.5 kN Yaw = 0.0°	T_2 = 32.3 kN Pitch = 0.0°	T_3 = 32.3 kN Roll = 0.0°
Result when $L_1 = L_2 = 22$ m	distance = 26.0 m	T_1 = 23.6 kN Yaw = -0.6°	T_2 = 32.4 kN Pitch = 7.66°	T_3 = 32.2 kN Roll = 11.53°

Physically measurable aircraft angle: $\textbf{Yaw} = \angle(\hat{\mathbf{n}}_x, \hat{\mathbf{c}}_y) - \frac{\pi}{2}$ ($\neq$ Euler yaw angle)

Yaw is the angle between $\hat{\mathbf{c}}_y$ and the vertical plane perpendicular to $\hat{\mathbf{n}}_x$. Yaw is positive when $\hat{\mathbf{c}}_y$ is directed backward ($\hat{\mathbf{c}}_y \cdot \hat{\mathbf{n}}_x < 0$, right turn) and is negative when $\hat{\mathbf{c}}_y$ is forward ($\hat{\mathbf{c}}_y \cdot \hat{\mathbf{n}}_x > 0$, left turn). Yaw is calculated as shown above, where $\angle(\hat{\mathbf{n}}_x, \hat{\mathbf{c}}_y)$ is the angle between $\hat{\mathbf{n}}_x$ and $\hat{\mathbf{c}}_y$.

Physically measurable aircraft angle: $\textbf{Pitch} = \angle(\hat{\mathbf{n}}_z, \hat{\mathbf{c}}_x) - \frac{\pi}{2}$ ($\neq$ Euler pitch angle)

Pitch is the angle between $\hat{\mathbf{c}}_x$ and Earth's horizon (the plane perpendicular to $\hat{\mathbf{n}}_z$). Pitch is regarded as positive when $\hat{\mathbf{c}}_x$ is directed upward ($\hat{\mathbf{c}}_x \cdot \hat{\mathbf{n}}_z < 0$) and is negative when $\hat{\mathbf{c}}_x$ is downward ($\hat{\mathbf{c}}_x \cdot \hat{\mathbf{n}}_z > 0$). Pitch is calculated as shown above, where $\angle(\hat{\mathbf{n}}_z, \hat{\mathbf{c}}_x)$ is the angle between $\hat{\mathbf{n}}_z$ (downward) and $\hat{\mathbf{c}}_x$.

Physically measurable aircraft angle: $\textbf{Roll} = \frac{\pi}{2} - \angle(\hat{\mathbf{n}}_z, \hat{\mathbf{c}}_y)$ ($\neq$ Euler roll angle)

Roll is the angle between $\hat{\mathbf{c}}_y$ and Earth's horizon. Roll is regarded as positive when $\hat{\mathbf{c}}_y$ is directed downward ($\hat{\mathbf{c}}_y \cdot \hat{\mathbf{n}}_z > 0$) and is negative when $\hat{\mathbf{c}}_y$ is upward ($\hat{\mathbf{c}}_y \cdot \hat{\mathbf{n}}_z < 0$).

21.10 Dynamics: Trifilar pendulum for an aircraft (builds on statics from Homework 21.9).

Create a dynamic simulation that allows you to vary constants and initial values so the aircraft of Hw 21.9 can be released from **rest** with **taut** cables from a physically meaningful location below the ceiling with -90° < yaw < 90°, -45° ≤ pitch ≤ 45°, -45° ≤ roll ≤ 45°. Add the following force and torque to damp the dynamic motion so it converges to your previous static equilibrium solution.

$$\vec{\mathbf{F}}^{C_{\rm cm}} = \text{-}b_F \; v^n \frac{{}^N\vec{\mathbf{v}}^{C_{\rm cm}}}{v + \epsilon_v} \qquad v = \left|{}^N\vec{\mathbf{v}}^{C_{\rm cm}}\right| \qquad\qquad \vec{\mathbf{T}}^{C} = \text{-}b_T \; w^n \frac{{}^N\vec{\boldsymbol{\omega}}^{C}}{w + \epsilon_w} \qquad w = \left|{}^N\vec{\boldsymbol{\omega}}^{C}\right|$$

Note: The damping constants b_F and b_T should be large enough so the motion damps quickly, but small enough to avoid overdamping the motion so the aircraft seems to be moving through sludge (integrator speed also slows). For this aircraft's mass and inertia, use $b_F = 6 \times 10^3$ and $b_T = 8 \times 10^3$. The exponent $n = \frac{1}{4}$ helps speed damping when $v < 1$ or $w < 1$. The constants $\epsilon_v = 1 \times 10^{-5} \, \frac{\rm m}{\rm s}$ and $\epsilon_w = 1 \times 10^{-5} \, \frac{\rm rad}{\rm s}$ help avoid divide-by-zero errors.

Track the system's mechanical energy, defined as $\boxed{\text{MechanicalEnergy} = K + U_{\rm gravity} - \text{W}_{\rm damping}}$ where: K is C's kinetic energy in N, $U_{\rm gravity}$ is C's gravitational potential energy in N, and $\text{W}_{\rm damping}$ is the work done on C by non-conservative forces.

Simulate: Use $L_1 = L_2 = L_3 = 30$ m and initial values $\text{yaw}_0 = 60°$, $\text{pitch}_0 = \text{roll}_0 = 0°$.

Result: (2^+ digits) **Simulation note:** With damping, the dynamic simulation converges to the static solution.

	distance	T_1	T_2	T_3	yaw	pitch	roll
Maximum value	30.0 m	102 kN	140 kN	140 kN	60°	0.73°	0.232°
Minimum value	22.4 m	5.4 kN	7.6 kN	7.5 kN	-54°	-0.12°	-0.13°

Verify: With initial values $\text{yaw}_0 = 0°$, $\text{pitch}_0 = \text{roll}_0 = 5°$, $C_{\rm cm}$ is **above** the ceiling by 21.304 cm.