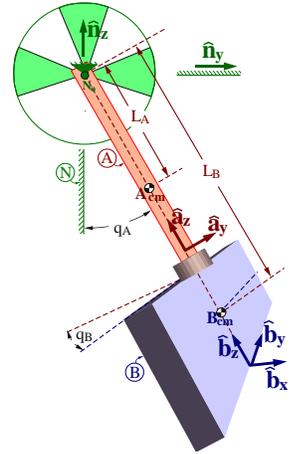


22.6.6 MG road-map: Chaotic motion of a double pendulum

The figure to the right is a schematic representation of a swinging babyboot attached by a shoelace to a rigid support. The mechanical model of the babyboot consists of a thin uniform rod A attached to a fixed support N by a revolute joint at point N_o and a uniform plate B connected to A with a second revolute joint at point B_o so B can rotate freely about A 's axis.

Note: The revolute joints' axes are *perpendicular*, not parallel.



Modeling considerations

- The plate, rod, and support are rigid.
- The revolute joints are ideal (massless, frictionless, no slop/flexibility).
- Earth is a Newtonian reference frame.
- Air resistance is negligible.
- Forces due to Earth's gravitation are uniform and constant.
- Other distance forces (electromagnetic and gravitational) are negligible.

Right-handed sets of unit vectors $\hat{n}_x, \hat{n}_y, \hat{n}_z$; $\hat{a}_x, \hat{a}_y, \hat{a}_z$; $\hat{b}_x, \hat{b}_y, \hat{b}_z$ are fixed in N, A, B , respectively, with $\hat{n}_x = \hat{a}_x$ parallel to the revolute axis joining A to N , \hat{n}_z vertically-upward, $\hat{a}_z = \hat{b}_z$ parallel to the rod's long axis (and the revolute axis joining B to A), and \hat{b}_z perpendicular to plate B .

Complete the following *MG road-map* for the angle q_A (angle from \hat{n}_z to \hat{a}_z with $+\hat{n}_x$ sense) and the angle q_B (angle from \hat{a}_y to \hat{b}_y with $+\hat{a}_z$ sense). Note: the "about points" are not unique.³

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point*	MG road-map equation
q_A				Draw		
q_B				Draw		
q_A	Dot(<input type="text"/> , System(<input type="text"/>).GetDynamics(<input type="text"/>))					MotionGenesis command ©
q_B	Dot(<input type="text"/> , <input type="text"/> .GetDynamics(<input type="text"/>))					MotionGenesis command ©

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³The shift theorem relates moment about B_{cm} and B_o as $\vec{M}^{B/B_{cm}} = \vec{M}^{B/B_o} + \vec{r}^{B_o/B_{cm}} \times \vec{F}^B$.
Since \vec{r}^{B_{cm}/B_o} is parallel to \hat{b}_z , $\hat{b}_z \cdot \vec{r}^{B_o/B_{cm}} \times \vec{F}^B = 0$, so $\hat{b}_z \cdot \vec{M}^{B/B_{cm}} = \hat{b}_z \cdot \vec{M}^{B/B_o}$.