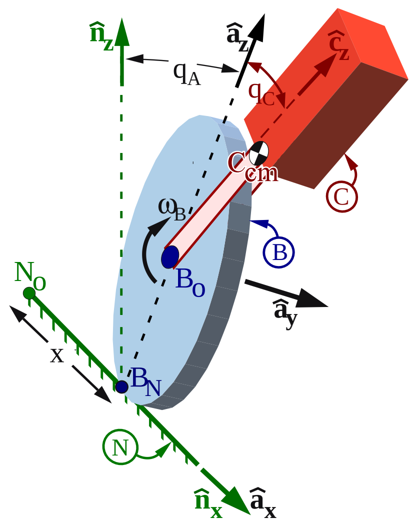
18.16 Bear riding a unicycle on a high-wire (Section 22.6).

Ernest the balancing bear
Purchase at Schylling toy



The figures to the right show a thin (massless) wheel B that rolls along a taut (rigid) cable N (a Newtonian reference frame). A rider and seat C attach to B with an ideal revolute joint at B_o (B's centroid). The revolute joint's axis is aligned with B's symmetry axis.

Right-handed orthogonal unit vectors $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ are fixed in N with $\hat{\mathbf{n}}_z$ vertically-upward and $\hat{\mathbf{n}}_x$ directed along the cable from a point N_o (fixed in N) to B_N (B's point of contact with N).

Right-handed orthogonal unit vectors $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ are fixed in a reference frame A with $\hat{\mathbf{a}}_x = \hat{\mathbf{n}}_x$ and $\hat{\mathbf{a}}_z$ directed from B_N to B_o.

Right-handed unit vectors $\hat{\mathbf{c}}_x$, $\hat{\mathbf{c}}_y$, $\hat{\mathbf{c}}_z$ are fixed in C and aligned with C's principal inertia axes for C_{cm} (C's center of mass). Their orientation in A is determined by first setting $\hat{\mathbf{c}}_i = \hat{\mathbf{a}}_i$ $(i = x, y, z)$ and then subjecting C to a right-handed rotation in A of $q_C\,\hat{\mathbf{a}}_y$.

Quantity	Symbol	Type	Value
Earth's gravitational constant	g	Constant	9.8 m/s^2
Radius of B	r_B	Constant	30 cm
$\hat{\mathbf{c}}_y$ measure of C_{cm}'s position vector from B_o	L_C	Constant	-35 cm
Mass of C	m^C	Constant	2 kg
C's central moment of inertia for $\hat{\mathbf{c}}_x$	I	Constant	3.4 kg m^2
C's central moment of inertia for $\hat{\mathbf{c}}_y$	J	Constant	3.2 kg m^2
C's central moment of inertia for $\hat{\mathbf{c}}_z$	K	Constant	2.8 kg m^2
$\hat{\mathbf{n}}_x$ measure of $\vec{\mathbf{r}}^{\,B_N/N_o}$	x	Variable	
Angle from $\hat{\mathbf{n}}_z$ to $\hat{\mathbf{a}}_z$ with $-\hat{\mathbf{n}}_x$ sense	q_A	Variable	
$\hat{\mathbf{a}}_y$ measure of ${}^A\vec{\boldsymbol{\omega}}^B$ (${}^A\vec{\boldsymbol{\omega}}^B = \omega_B\,\hat{\mathbf{a}}_y$)	ω_B	Variable	
Angle from $\hat{\mathbf{a}}_z$ to $\hat{\mathbf{c}}_z$ with $+\hat{\mathbf{a}}_y$ sense	q_C	Variable	

Complete the following ***MG road-map*** for this systems's equations of motion.

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	***MG road-map equation***	Additional Unknowns
				Draw			*
				Draw			
				Draw			
				Draw			

* Additional constraint equation: whose time-derivative is:

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Optional: Consider a bear that pedals so the $\hat{\mathbf{a}}_y$ measure of the torque on B from C is

$$T^{B/C} = -0.3\,(x - x_{\text{Desired}}) - 0.6\,\dot{x} \qquad \text{Control system to pedal the unicycle to } x_{\text{Desired}} = 10 \text{ m.}$$

Plot x, q_A, q_C for $0 \le t \le 12$ seconds. Use initial values $x = 0$ m, $q_A = 10°$, $q_B = 0°$, $\dot{x} = \dot{q}_A = \dot{q}_B = 0$.

Result: