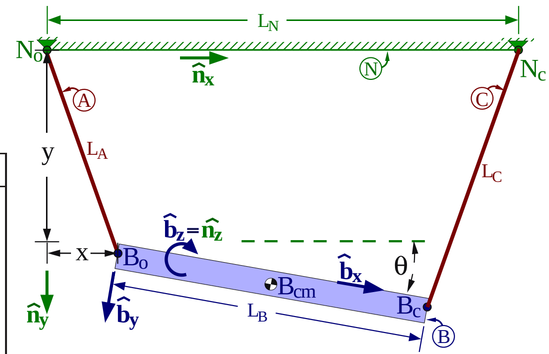
22.4 Statics & dynamics for a construction hoist (or beam) on two cables (se Hw 4.15, 8.13).

A rigid uniform beam B is attached to a flat horizontal roof N by two inextensible cables A and C.
Cable A attaches to the roof at point N_o of N and to the beam at point B_o of B.
Cable C attaches to the roof at point N_C of N and to the beam at point B_C of B.
Right-handed sets of orthogonal unit vectors $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ and $\hat{\mathbf{b}}_x$, $\hat{\mathbf{b}}_y$, $\hat{\mathbf{b}}_z$ are fixed in N and B, with:

- $\hat{\mathbf{n}}_x$ horizontally-right from N_o to N_C.
- $\hat{\mathbf{n}}_y$ vertically-downward (gravity direction).
- $\hat{\mathbf{b}}_x$ from B_o to B_C and $\hat{\mathbf{b}}_z = \hat{\mathbf{n}}_z$ perpendicular to the plane containing points N_o, B_o, B_{cm}, B_C, N_C.



Description	Symbol	Value
Distance between N_o and N_C	L_N	6 m
Distance between B_o and B_C	L_B	4 m
Length of cable A	L_A	2.7 m
Length of cable C	L_C	3.7 m
Mass of beam B	m	120 kg
B's moment of inertia about B_{cm} for $\hat{\mathbf{n}}_z$	I_{zz}	$\frac{1}{12}\, m\, L_B^2$
Earth's gravitational acceleration	g	$9.8\ \text{m/s}^2$
Damping constant in force $-b\,{}^N\vec{\mathbf{v}}^{B_{cm}}$	b	230 N∗s/m
$\hat{\mathbf{n}}_x$ measure of B_o's position from N_o	x	Variable
$\hat{\mathbf{n}}_y$ measure of B_o's position from N_o	y	Variable
Angle from $\hat{\mathbf{n}}_x$ to $\hat{\mathbf{b}}_x$ with $^+\hat{\mathbf{n}}_z$ sense	θ	Variable

Regard this is a **1** degree-of-freedom system. Without the 2 cables, the beam has 3 degrees-of-freedom in the plane (2 translational, 1 rotational), or more generally 6 degrees-of-freedom in space (3 translational, 3 rotational).

(a) Form this system's statics and dynamics equations with one or more of the following.

Type	***Kane's method***	***Lagrange's method***	***FBD***
Augmented	***generalized speeds*** $\dot{x}, \dot{y}, \dot{\theta}$	***generalized coordinates*** x, y, θ	x, y, θ
Augmented/Embedded	***generalized speeds*** $\dot{x}, \dot{\theta}$	***generalized coordinates*** x, θ	No
Embedded	***generalized speed*** x	No	No

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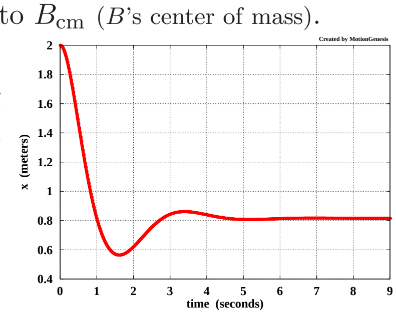
(b) Using the statics equations, determine the physically meaningful static equilibrium solution for x, y, θ. **Optional:** Determine the tensions T_A and T_C in cables A and C.
Result: $x = 0.815$ m $\quad y = 2.574$ m $\quad \theta_B = 12.934°$ $\quad T_A = 665.19$ N $\quad T_C = 577.90$ N

(c) Simulate with an aerodynamic damping force $-b\,{}^N\vec{\mathbf{v}}^{B_{cm}}$ applied to B_{cm} (B's center of mass).

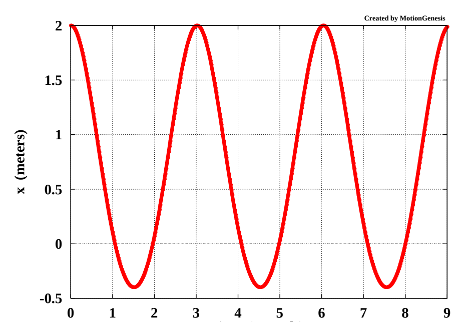
Knowing $x(0) = 2$ m, the initial values for y and θ are calculated (from Hw 4.15) as $y(0) = 1.813836$ m, $\theta(0) = 27.674°$. Knowing $\dot{x}(0) = 0$, $\dot{y}(0) = \dot{\theta}(0) = 0$. Numerically integrate the ODEs and graph $x(t)$ for $0 \le t \le 9$. Determine x, y, θ at $t = 9$ sec.



Result: $x(9) = 0.815$ m $\quad y(9) = 2.574$ m $\quad \theta(9) = 12.93°$
At $t = 9$ sec, this dynamic solution approximates the static solution to two significant digits **True**/**False**.

(d) Simulate the beam's motion again with no damping ($b = 0$).

Graph $x(t)$ for $0 \le t \le 9$ and approximate the period of undamped oscillations of $x(t)$.



Result: $\quad \tau_{\text{period}} \approx 3.02$ sec

†Optional: Linearize the dynamics about the static solution. Verify the estimated value for τ_{period} with the the value predicted by the linearized equations of motion.
Result: $\quad 107.4\,\ddot{\tilde{x}} + 197.75\,\dot{\tilde{x}} + 465.33\,\tilde{x}$