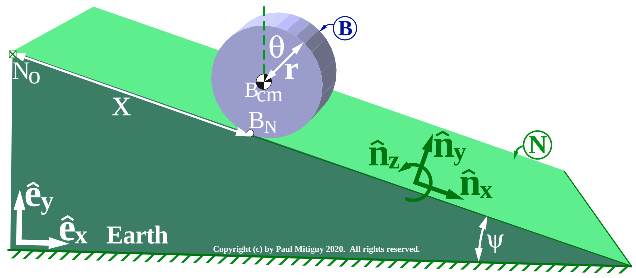
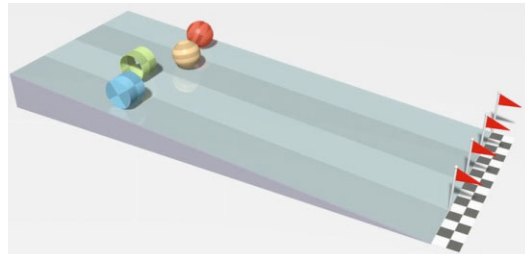
Energy of a disk on an inclined plane.

A disk B slides/rolls on a rough ($\mu_{\text{friction}} \neq 0$) inclined plane N. The plane is inclined at angle ψ with Earth E. Right-handed orthogonal unit vectors $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ and $\widehat{\mathbf{e}}_x$, $\widehat{\mathbf{e}}_y$, $\widehat{\mathbf{e}}_z$ are fixed in N and E respectively, with $\widehat{\mathbf{n}}_x$ directed downhill, $\widehat{\mathbf{n}}_y$ perpendicular to the plane, $\widehat{\mathbf{e}}_z = \widehat{\mathbf{n}}_z$ parallel to B's symmetry axis, and $\widehat{\mathbf{e}}_y$ vertically upward.



Quantity	Symbol	Type
Radius of B	r	Constant
Mass of B	m	Constant
B's moment of inertia about B_{cm} for $\widehat{\mathbf{n}}_z$	I_{zz}	Constant
Earth's gravitational constant	g	Constant
Inclined plane angle with Earth's horizontal	ψ	Constant
$\widehat{\mathbf{n}}_x$ measure of B_N's position from N_o	x	Variable
B's rotation angle from $\widehat{\mathbf{e}}_y$ (with $-\widehat{\mathbf{n}}_z$ sense)	θ	Variable



(a) Determine B's angular velocity in N and the velocity of B_{cm} (B's center of mass) in N. Denoting B_N as the point of B in contact with N at each instant, form B_N's velocity in N.
Result: ${}^N\vec{\boldsymbol{\omega}}^B = \square$ ${}^N\vec{\mathbf{v}}^{B_{cm}} = \square\, \widehat{\mathbf{n}}_x$ ${}^N\vec{\mathbf{v}}^{B_N} = (\square - \square)\,\square$

(b) Now assume B **rolls** on N. Use the definition of rolling to determine another expression for ${}^N\vec{\mathbf{v}}^{B_N}$, and then use it to express $\dot{x}$ in terms of r and $\dot{\theta}$.
Result: ${}^N\vec{\mathbf{v}}^{B_N} \underset{\text{(rolling)}}{=} \square \quad \Rightarrow \quad \dot{x} = \square$

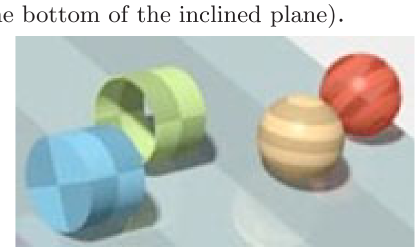
(c) Form K (B's kinetic energy in N) when B **slides** or **rolls** on N in terms of m, r, I_{zz}, $\dot{\theta}$, $\dot{x}$.
Result: Sliding: $K = \frac{1}{2}\,\square\,\dot{x}^2 + \frac{1}{2}\,\square$ Rolling: $K = \frac{1}{2}\,(\square + \square)\,\dot{\theta}^2$

(d) Assuming B **rolls** on N and the only external forces on B are contact forces at B_N and Earth's gravity, form B's power P in N, **a** potential energy U for forces on B, and decide if $P = -\frac{dU}{dt}$.
Result: $P \underset{(22.2)}{\triangleq} \sum \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = \square\,\dot{x}$ $U = \square\, x + \text{arbitraryConstant}$ $P = -\frac{dU}{dt}$? **Yes/No**

(e) When B **slides** on N, $K + U = K_{\text{initial}} + U_{\text{initial}}$ (***conserves mechanical energy***) **True/False**.
When B **rolls** on N, $K + U = K_{\text{initial}} + U_{\text{initial}}$ (***conserves mechanical energy***) **True/False**.

(f) Each uniform-density object below has the same mass m and radius r. Report formulas for each object's I_{zz} in terms of m and r (use the Internet or a textbook's inertia appendix). For the solid disk, knowing B **rolls** and initial values are $x(0) = 0$ and $\dot{\theta}(0) = 0$, express $\dot{\theta}$ in terms of g, r, x. Label the objects ***1st***, ***2nd***, ***3rd***, ***4th*** (in the order that objects reach the bottom of the inclined plane).

Solid disk	Hollow disk	Solid sphere	Hollow sphere
$I_{zz} = \frac{1}{\square}\, m\, r^2$	$I_{zz} = m\, r^2$	$I_{zz} = \frac{2}{\square}\, m\, r^2$	$I_{zz} = \frac{2}{3}\, m\, r^2$
$\dot{\theta} = \sqrt{\frac{\square}{\square}}$	$\dot{\theta} = \sqrt{\frac{g\,\sin(\psi)\,x}{r^2}}$	$\dot{\theta} = \sqrt{\frac{10\,g\,\sin(\psi)\,x}{7\,r^2}}$	$\dot{\theta} = \sqrt{\frac{6\,g\,\sin(\psi)\,x}{5\,r^2}}$
$\square$	***4th***	$\square$	$\square$



Animation/results at **www.MotionGenesis.com** ⇒ **Get Started** ⇒ **Rolling disks.**

(g) A force is required to enforce a rolling constraint. Based on your physical experience, circle the type of force that best describes the reason a bicycle wheel rolls instead of slips/slides.

Gravity	**Air resistance**	**Magnetics**	**Inertia**	**Friction**	**Damping**	**Spring**	**Buoyancy**

Note: Without this force, it is very difficult/dangerous to ride a bike (e.g., on smooth ice).