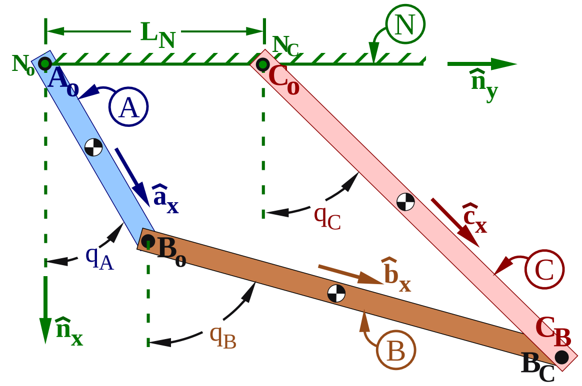
21.2 Four-bar linkage statics (refers to Hw 10.7).

The figure to the right shows a planar four-bar linkage consisting of frictionless-pin-connected uniform rigid links A, B, and C and ground N.

- Link A connects to N and B at points A_o and A_B
- Link B connects to A and C at points B_o and B_C
- Link C connects to N and B at points C_o and C_B
- Point N_o of N is coincident with A_o
- Point N_C of N is coincident with C_o



Right-handed orthogonal unit vectors $\hat{\mathbf{a}}_i$, $\hat{\mathbf{b}}_i$, $\hat{\mathbf{c}}_i$, $\hat{\mathbf{n}}_i$ $(i = x, y, z)$ are fixed in A, B, C, N, with:

- $\hat{\mathbf{a}}_x$ directed from A_o to A_B
- $\hat{\mathbf{b}}_x$ directed from B_o to B_C
- $\hat{\mathbf{c}}_x$ directed from C_o to C_B
- $\hat{\mathbf{n}}_x$ vertically-downward
- $\hat{\mathbf{n}}_y$ directed from N_o to N_C
- $\hat{\mathbf{a}}_z = \hat{\mathbf{b}}_z = \hat{\mathbf{c}}_z = \hat{\mathbf{n}}_z$ parallel to pin axes

Quantity	Symbol	Value
Length of link A	L_A	1 m
Length of link B	L_B	2 m
Length of link C	L_C	2 m
Distance between N_o and N_C	L_N	1 m
Mass of A	m^A	10 kg
Mass of B	m^B	20 kg
Mass of C	m^C	20 kg
Earth's gravitational acceleration	g	9.81 $\frac{m}{s^2}$
$\hat{\mathbf{n}}_y$ measure of force applied to C_B	H	200 N
Angle from $\hat{\mathbf{n}}_x$ to $\hat{\mathbf{a}}_x$ with $+\hat{\mathbf{n}}_z$ sense	q_A	Variable
Angle from $\hat{\mathbf{n}}_x$ to $\hat{\mathbf{b}}_x$ with $+\hat{\mathbf{n}}_z$ sense	q_B	Variable
Angle from $\hat{\mathbf{n}}_x$ to $\hat{\mathbf{c}}_x$ with $+\hat{\mathbf{n}}_z$ sense	q_C	Variable

As in Hw 10.7, create the following "***loop equation***" and dot-product with $\hat{\mathbf{n}}_x$ and $\hat{\mathbf{n}}_y$.

$$L_A\,\hat{\mathbf{a}}_x + L_B\,\hat{\mathbf{b}}_x - L_C\,\hat{\mathbf{c}}_x - L_N\,\hat{\mathbf{n}}_y = \vec{0}$$

Complete the following ***MG road-map*** to determine this systems's ***static configuration***.

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point*	*MG road-map equation*	Additional Unknowns
q_A	Rotate	$\hat{\mathbf{n}}_z$	A, B	**Draw**	A_o	$\hat{\mathbf{a}}_x \cdot \vec{\mathrm{M}}^{S/A_o} = 0$	F_x^C, F_y^C
q_B	Rotate	$\hat{\mathbf{n}}_z$	B	**Draw**	B_o	$\hat{\mathbf{a}}_y \cdot \vec{\mathrm{M}}^{B/B_o} = 0$	F_x^C, F_y^C
q_C	Rotate	$\hat{\mathbf{n}}_z$	C	**Draw**	C_o	$\hat{\mathbf{a}}_y \cdot \vec{\mathrm{M}}^{C/C_o} = 0$	F_x^C, F_y^C

* Additional constraint equation: $-L_A \sin(q_A)\,\dot{q}_A - L_B \sin(q_B)\,\dot{q}_B + L_C \sin(q_C)\,\dot{q}_C = 0$

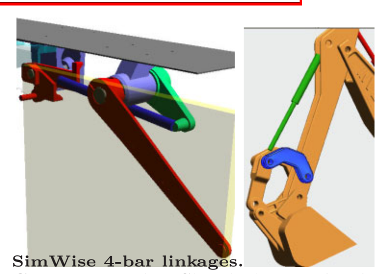
* Additional constraint equation: $L_A \cos(q_A)\,\dot{q}_A + L_B \cos(q_B)\,\dot{q}_B - L_C \cos(q_C)\,\dot{q}_C = 0$

q_A	**Dot(Nz> , System(A, B).GetStatics(Ao))**	**M**otion**G**enesis command ©
q_B	**Dot(Nz> , B .GetStatics(Bo))**	**M**otion**G**enesis command ©
q_C	**Dot(Nz> , C.GetStatics(Co))**	**M**otion**G**enesis command ©

Determine the ***static equilibrium*** values of q_A, q_B, q_C.
Use your intuition (**guess**), circle the ***stable*** solution.

Solution 1	$q_A \approx 20.0°$	$q_B \approx 71.7°$	$q_C = 38.3°$
Solution 2	$q_A \approx 249.3°$	$q_B \approx 140.2°$	$q_C = 199.1°$
Solution 3	$q_A \approx 30.7°$	$q_B \approx 226.1°$	$q_C = 254.7°$

Solution at **www.MotionGenesis.com** ⇒ **Get Started** ⇒ **Four-bar linkage**



SimWise 4-bar linkages.
Courtesy Design Simulation Technology