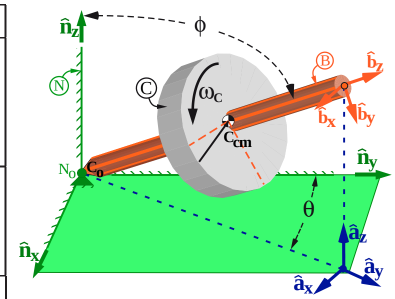
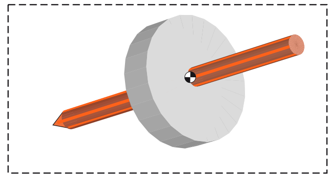
## Dynamic simulation: Precessing, nutating, spinning gyro

$$\vec{\mathbf{H}} = \vec{\vec{\mathbf{I}}} \cdot \vec{\boldsymbol{\omega}} \qquad \vec{\mathbf{M}} = \frac{^N d\, \vec{\mathbf{H}}}{dt}$$

| Quantity | Symbol | Type | (Initial) Value |
|---|---|---|---|
| Earth's gravitational constant | $g$ | Constant | $9.8\ \text{m/s}^2$ |
| Mass of $C$ | $m$ | Constant | $0.1$ kg |
| Radius of $C$ | $r$ | Constant | $0.2$ m |
| Distance from $N_o$ to $C_{cm}$ | $L$ | Constant | $0.2$ m |
| Angle from $\hat{\mathbf{n}}_y$ to $\hat{\mathbf{a}}_y$ with $-\hat{\mathbf{n}}_z$ sense | $\theta$ | Variable | $0°$ |
| Angle from $\hat{\mathbf{a}}_z$ to $\hat{\mathbf{b}}_z$ with $-\hat{\mathbf{a}}_x$ sense | $\phi$ | Variable | $20°$ |
| "Rotor spin" $\,{}^B\vec{\boldsymbol{\omega}}^C \cdot \hat{\mathbf{b}}_z$ | $\omega_C$ | Variable | 300 rpm |
| Note: $\dot{\theta}(t=0) = 0$ and $\dot{\phi}(t=0) = 0$ | | | |




(a) Assuming $B$ is in single-point contact with $N$, draw a ***free-body diagram (FBD)*** of the system $S$ formed by $B$ and $C$. Determine the moment of all contact and distance forces on $S$ about $N_o$.
**Result:** $\vec{\mathbf{M}}^{S/N_o} = {}^{N_o}\vec{\mathbf{r}}^{C_{cm}} \times (-m\, g\, \hat{\mathbf{n}}_z) = -m\, g\, L\, \sin(\phi)\, \hat{\mathbf{b}}_x$



(b) Using results from part (a) and Homework 15.17, form three scalar equations governing the motion of $S$ in $N$ by equating the moment of forces on $S$ about $N_o$ to the time-derivative in $N$ of $S$'s angular momentum about $N_o$ in $N$. Next, solve these equations for $\ddot{\theta}$, $\ddot{\phi}$, $\dot{\omega}_C$.
**Result:**

$$\boxed{\vec{\mathbf{M}}^{S/N_o} \underset{(22.4)}{=} \frac{^N d\ {}^N\vec{\mathbf{H}}^{S/N_o}}{dt}} \Rightarrow$$

$$\ddot{\theta} = -2\, \dot{\phi}\, \frac{r^2\, \omega_C + 4\, L^2 \cos(\phi)\, \dot{\theta}}{(r^2 + 4\, L^2)\, \sin(\phi)}$$

$$\ddot{\phi} = \sin(\phi)\, \frac{(4\, L^2 - r^2)\cos(\phi)\, \dot{\theta}^2 + 2\, r^2\, \omega_C\, \dot{\theta} + 4\, g\, L}{r^2 + 4\, L^2}$$

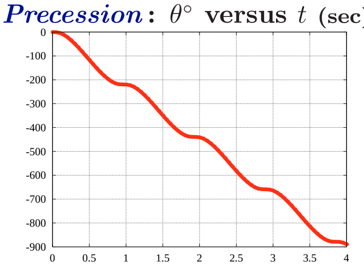
$$\dot{\omega}_C = -\dot{\phi}\left\{\sin(\phi)\, \dot{\theta} + \frac{2\left[r^2\, \omega_C + 4\, L^2 \cos(\phi)\, \dot{\theta}\right]}{(r^2 + 4\, L^2)\tan(\phi)}\right\}$$

**Consider using MotionGenesis**

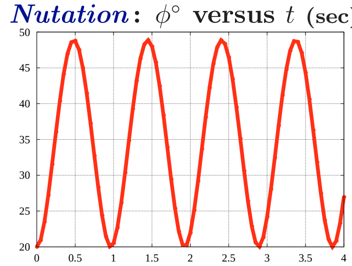
**Optional:** Try ***Lagrange's equations*** (Chapter 27) with $\dot{\theta}_C = \omega_C$.
Or use Kane's method in Chapter 26.

(c) Solve the differential equations for $\theta(t)$, $\phi(t)$, $\omega_C(t)$ with the given initial values (see table above) and a numerical integration step of 0.05 sec. Generate plots of $\theta(t)$ and $\phi(t)$ for $0 \le t \le 4$ sec.
www.MotionGenesis.com ⇒ Get Started ⇒ Gyro

***Precession*:** $\theta°$ **versus** $t$ **(sec)**




***Nutation*:** $\phi°$ **versus** $t$ **(sec)**




(d) For $0 \le t \le 4$ sec, plot (or view output data for) $H_{\hat{\mathbf{n}}_z} = {}^N\vec{\mathbf{H}}^{C/N_o} \cdot \hat{\mathbf{n}}_z$ and $H_{\hat{\mathbf{b}}_z} = {}^N\vec{\mathbf{H}}^{C/N_o} \cdot \hat{\mathbf{b}}_z$.
Knowing a potential energy for $S$ in $N$ is $\text{U} = m\, g\, L \cos(\phi)$, also plot (or view output for) mechanical energy $= \text{K} + \text{U}$ (the sum of $S$'s kinetic energy and potential energy in $N$).
To numerical integrator accuracy, circle the following quantities that remain constant.

| | | |
|---|---|---|
| $H_{\hat{\mathbf{n}}_z}$ | $H_{\hat{\mathbf{b}}_z}$ | Mechanical energy |

(e) An engineer would like to model damping with a torque on $C$ from $B$ of $T_z\, \hat{\mathbf{b}}_z$. Assuming $B$ is massless and is in either single-point contact with $N$ along $B$'s symmetrical axis or $B$ is connected to $N$ by a frictionless ball-and-socket joint, is it possible for $T_z \neq 0$? **Yes/No**.

**Draw a *FBD* of** $B$ **and explain.**