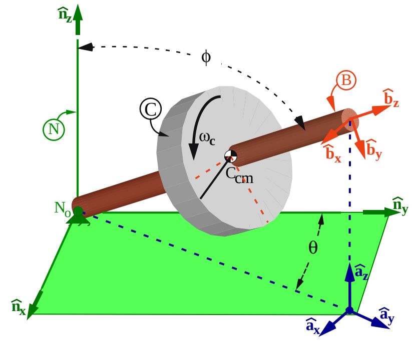
11.12 Motion of a precessing, nutating, spinning, gyro (refers to Homework 11.11).

Quantity	Symbol	Type	(Initial) Value
Earth's gravitational constant	g	Constant	$9.8\ \text{m/s}^2$
Mass of C	m	Constant	0.1 kg
Radius of C	r	Constant	0.2 m
Distance from N_o to C_{cm}	L	Constant	0.2 m
Angle between $\widehat{\mathbf{n}}_y$ and $\widehat{\mathbf{a}}_y$	θ	Variable	$0°$
Angle between $\widehat{\mathbf{a}}_z$ and $\widehat{\mathbf{b}}_z$	ϕ	Variable	$20°$
${}^{B}\vec{\boldsymbol{\omega}}^{C} \cdot \widehat{\mathbf{b}}_z$	ω_C	Variable	300 rpm
Note: $\dot{\theta}(t=0) = 0$ and $\dot{\phi}(t=0) = 0$			



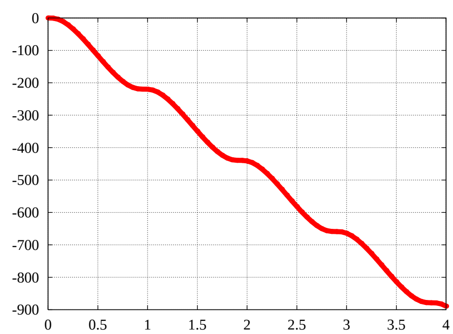
(a) Consider the system S formed by B and C. Express the moment of all contact and distance forces acting on S about N_o in terms of $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$.
Result:

$$\vec{\mathbf{M}}^{S/N_o} = \vec{\mathbf{r}}^{C_{cm}/N_o} \times (\text{-}m\,g\,\widehat{\mathbf{n}}_z) = \text{-}m\,g\,L\,\sin(\phi)\,\widehat{\mathbf{b}}_x$$

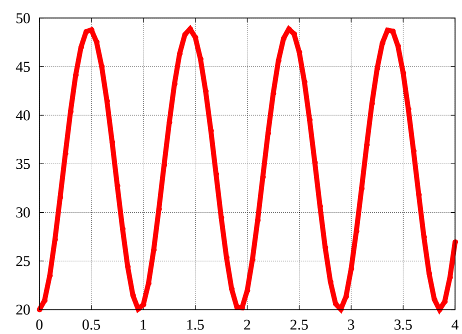
(b) Form three scalar equations governing the motion of S in N by equating the moment of applied forces on S about N_o to the time-derivative in N of S's angular momentum about N_o in N. Next, solve these equations for $\ddot{\theta}$, $\ddot{\phi}$, and $\dot{\omega}_C$.
Result:

$$\boxed{\vec{\mathbf{M}}^{S/N_o} \underset{(10.??)}{=} \frac{{}^{N}d\ {}^{N}\vec{\mathbf{H}}^{S/N_o}}{dt}} \quad \Rightarrow \quad
\begin{aligned}
\ddot{\theta} &= \text{-}2\,\dot{\phi}\ \frac{r^2\,\omega_C + 4\,L^2\,\cos(\phi)\,\dot{\theta}}{(r^2 + 4\,L^2)\,\sin(\phi)} \\
\ddot{\phi} &= \sin(\phi)\ \frac{(4\,L^2 - r^2)\,\cos(\phi)\,\dot{\theta}^2 \ +\ 2\,r^2\,\omega_C\,\dot{\theta}\ +\ 4\,g\,L}{r^2\ +\ 4\,L^2} \\
\dot{\omega}_C &= \text{-}\dot{\phi}\ \left\{ \sin(\phi)\,\dot{\theta}\ +\ \frac{2\,[r^2\,\omega_C + 4\,L^2\,\cos(\phi)\,\dot{\theta}]}{(r^2 + 4\,L^2)\,\tan(\phi)} \right\}
\end{aligned}$$

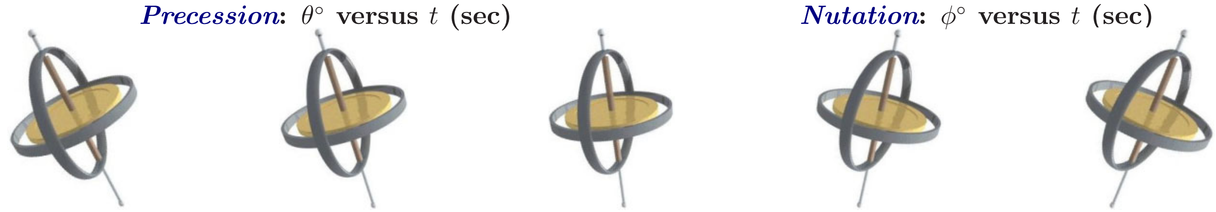
(c) Solve the differential equations for $\theta(t)$, $\phi(t)$, $\omega_C(t)$ with the given initial values (see table above) using a numerical integration step of 0.05 sec. Generate plots of $\theta(t)$ and $\phi(t)$ for $0 \le t \le 4$ sec.
Note: **M**otion**G**enesis solves this problem at www.MotionGenesis.com $\Rightarrow$ Get Started $\Rightarrow$ Gyro.



Precession: $\theta°$ **versus** t **(sec)**



Nutation: $\phi°$ **versus** t **(sec)**



(d) Calculate time-histories for $0 \le t \le 4$ sec of $H_{\widehat{\mathbf{n}}_z} \triangleq {}^{N}\vec{\mathbf{H}}^{C/N_o} \cdot \widehat{\mathbf{n}}_z$ and $H_{\widehat{\mathbf{b}}_z} \triangleq {}^{N}\vec{\mathbf{H}}^{C/N_o} \cdot \widehat{\mathbf{b}}_z$.
Knowing a potential energy for S in N is $U^S = m\,g\,L\,\cos(\phi)$, calculate the time-histories of the mechanical energy $KePe$ (the sum of S's kinetic energy and potential energy in N).
To numerical integrator accuracy, the numerical solution shows the value of $H_{\widehat{\mathbf{n}}_z}$ is $\boxed{\text{constant}}$**/variable,** $H_{\widehat{\mathbf{b}}_z}$ is $\boxed{\text{constant}}$**/variable,** and mechanical energy is $\boxed{\text{constant}}$**/variable.**