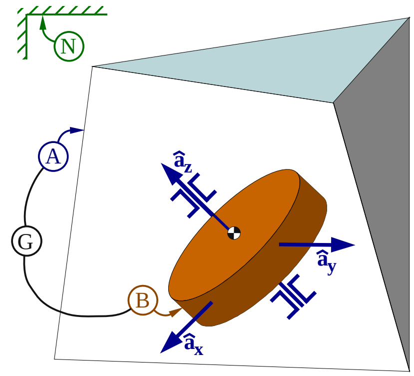
9.23 Gyrostat spin stabilization



The figure to the right shows a gyrostat `G` consisting of a carrier A and a thin uniform cylindrical rotor B moving in a Newtonian frame N. Dextral sets of orthogonal unit vectors $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ are fixed in A and are parallel to G's central principal inertia axes. The rotor B has a central moment of inertia of J about its symmetric axes, which is parallel to $\hat{\mathbf{a}}_z$. G's central principal moments of inertia for $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ are denoted I_{xx}, I_{yy}, I_{zz}, respectively. The generalized speeds ω_x, ω_y, ω_z are the $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ measures of A's angular velocity in N. The constant Ω is the $\hat{\mathbf{a}}_z$ measure of B's angular velocity in A.

When numerical values are required, use $J = 0.07634 \text{ kg m}^2$, $I_{xx} = 1.25 \text{ kg m}^2$, $I_{yy} = 4.25 \text{ kg m}^2$, $I_{zz} = 5 \text{ kg m}^2$, $\omega_{z\text{nom}} = 1 \frac{\text{rad}}{\text{sec}}$.

- Form equations of motion which govern angular motions of the system.
  Result:

$$I_{xx}\,\dot{\omega}_x + \omega_y\,[J\,\Omega - (I_{yy} - I_{zz})\,\omega_z] = 0$$
$$I_{yy}\,\dot{\omega}_y - \omega_x\,[J\,\Omega - (I_{xx} - I_{zz})\,\omega_z] = 0$$
$$I_{zz}\,\dot{\omega}_z - (I_{xx} - I_{yy})\,\omega_x\,\omega_y = 0$$

- Consider the nominal solution `wx = wy = wx' = wy' = wz' = 0, wz = nwz` where `nwz` is a constant. After introducing the variables `dwx`, `dwy`, `dwz` as perturbations of ω_x, ω_y, ω_z, linearize the equations of motion in the perturbations about the nominal solution. Put the linearized equations in the form `X' = A*X` where `A` is a 3×3 coefficient matrix and `X` is the 3×1 state matrix `[dwx; dwy; dwz]`.
  Result:

$$A = \begin{bmatrix} 0 & -\frac{J\,\Omega - (I_{yy}-I_{zz})\,\omega_{z\text{nom}}}{I_{xx}} & 0 \\ \frac{J\,\Omega - (I_{yy}-I_{zz})\,\omega_{z\text{nom}}}{I_{yy}} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

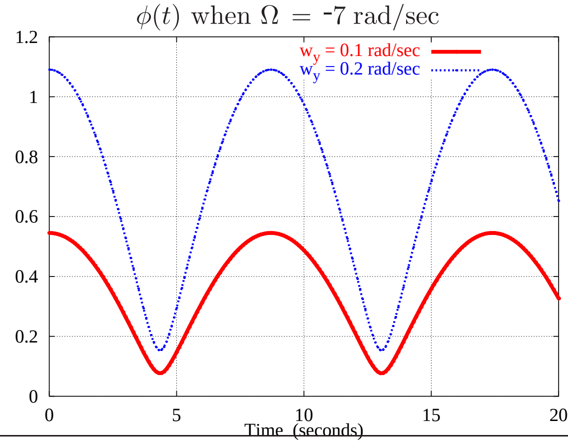
- Determine values of Ω which result in eigenvalues λ of A that are positive.
  Result: The **M**otion**G**enesis response $0.0011\,(9.824 + \Omega)\,(49.12 + \Omega) + \lambda^2 = 0$ shows

λ^2 is positive when	$-49.12 < \Omega < -9.82$	$\text{Real}(\lambda) > 0$	so solution is "**unstable**"
λ^2 is negative when	$\Omega > -9.82$ or $\Omega < -49.12$	$\text{Real}(\lambda) = 0$	so solution is "**stable**"

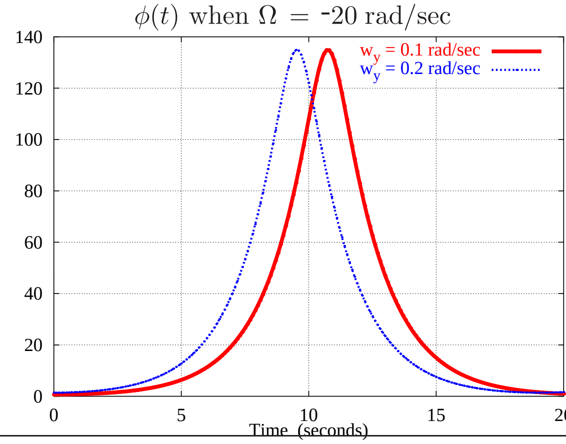
- Using the nonlinear equations of motion, run four 20 sec simulations, all with $\omega_x = 0$ and $\omega_z = 1$. Plot the time-history of ϕ, the angle between $\hat{\mathbf{a}}_z$ and the inertial angular momentum of G. For each simulation, check that H (the magnitude of the gyrostats's angular momentum in N) is time-invariant.
  Result: See the plots. By examining (or plotting) the simulation output file `MGGyrostatSpinStability.1`, it is clear that `H` is time-invariant.

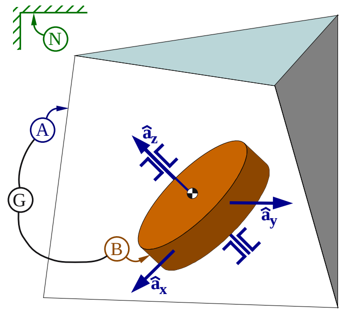
#	Ω	ω_y
A	-7	0.02
B	-7	0.01
C	-20	0.02
D	-20	0.01

$\phi(t)$ when $\Omega = -7$ rad/sec



$\phi(t)$ when $\Omega = -20$ rad/sec





```
% File: MGGyrostatSpinStability.txt
% Copyright (c) Motion Genesis LLC.  All rights reserved.
%--------------------------------------------------------------------
NewtonianFrame  N          % Newtonian reference frame
RigidBody       A          % Carrier
RigidFrame      B          % Rotor
%--------------------------------------------------------------------
Constant  Omega = -20 rad/sec  % B's' angular speed  in A
Constant  J = 0.07634 kg*m^2   % B's moment of inertia about spin axis
Variable  wx',  wy',  wz'
SetGeneralizedSpeed( wx, wy, wz )
%--------------------------------------------------------------------
%       Mass and inertia   (attribute G's inertia to A)
A.SetMassInertia( m,  Ix = 1.25 kg*m^2,  Iy = 4.25 kg*m^2,  Iz = 5 kg*m^2 )
%--------------------------------------------------------------------
%       Angular velocities
A.SetAngularVelocityAcceleration( N, wx*Ax> + wy*Ay> + wz*Az> )
B.SetAngularVelocityAcceleration( A, Omega*Az> )
%--------------------------------------------------------------------
%       Velocity and acceleration of G's mass center is 0>
Acm.SetVelocityAcceleration(  N,  0>  )
%--------------------------------------------------------------------
%       Equations of motion
Zero = System.GetGeneralizedForce() + Frstar() + Gyrostat(FrStar,CYLINDER,A,B,J)
%--------------------------------------------------------------------
%       Calculate gyrostat's angular momentum
H> = System.GetAngularMomentum(Acm) + Gyrostat(Angmom,CYLINDER,A,B,J)
H = GetMagnitude( H> )
%--------------------------------------------------------------------
%       Angle between angular momentum vector H> and Az>
phi = AngleBetweenUnitVectors(  GetUnitVec( H> ),  Az> )
%--------------------------------------------------------------------
%       Integration parameters and values for constants and variables
Input  tFinal = 20,  tStep = 0.1,  absError = 1.0E-07
Input  wx = 0 rad/sec,  wy = 0.02 rad/sec,  wz = 1 rad/sec
%--------------------------------------------------------------------
%       Quantities to be output by ODE command.
Output  t sec,  phi degs,  H kg*m^2/s
ODE( Zero,  wx', wy', wz' )  MGGyrostatSpinStability
%********************************************************************
%       STABILITY ANALYSIS
%********************************************************************
%       Linearization: Perturbation vars + nominal solution parameters.
Variable  dwx',  dwy',  dwz'      % Perturbations of wx, wy, wz.
Constant  nwz = 1 rad/sec         % Nominal solution for wz.
%--------------------------------------------------------------------
%       Check nominal solution satisifies the equations of motion.
Check = Evaluate( Zero,  wx=0, wx'=0,  wy=0, wy'=0,  wz=nwz, wz'=0 )
%--------------------------------------------------------------------
%       Linearize equations of motion about nominal solution.
Perturb = Linearize1( Zero, wx = 0 : dwx,  wx' = 0 : dwx',  wy = 0 : dwy, &
                            wy' =0 : dwy', wz = nwz: dwz,  wz' = 0 : dwz' )
Solve( Perturb,  dwx', dwy', dwz' )
%--------------------------------------------------------------------
%       Form, X, X', and A matrices in the matrix equation X' = A * x
Xm = [ dwx;  dwy;  dwz ]
Xp = Dt( Xm )
Am = D(  Xp,  Transpose(Xm)  )
%--------------------------------------------------------------------
%       To find eigenvalues of Am symbolically, find the roots of the
%       equation found by setting determinant( Lambda * I - A ) = 0.
Variable Lambda
det = Determinant( Lambda * GetIdentityMatrix(3) - Am )
det /= Lambda                     % Inspection of det shows Lambda = 0 is a root
%--------------------------------------------------------------------
%       Find values of Omega which result in Lambda > 0.
det := EvaluateAtInput( det,  Omega = Omega )
%--------------------------------------------------------------------
Save MGGyrostatSpinStability.all
Quit
```

Note: Problem solution at **www.MotionGenesis.com** ⇒ **Get Started** ⇒ **Gyros**.

F=ma