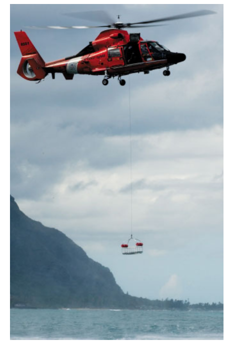
11.16 Helicopter retrieval motion & simulation. $\vec{\mathbf{F}} = m\vec{\mathbf{a}} \;\Rightarrow\; \ddot{\theta} \;\Rightarrow\; \dot{\theta} \;\Rightarrow\; \theta$

A helicopter A retrieves capsized fishermen in a rescue bucket that is ***rigidly attached*** to the distal end of a relatively light straight cable B (the fishermen and rescue bucket are modeled as a particle Q). The cable's length ***changes*** due to a retrieval motor at point A_o of helicopter A.



Description	Symbol	Type	Value or specification
Mass of Q (fishermen and bucket)	m	Constant	100 kg
Earth's gravitational acceleration	g	Constant	$9.8\ \frac{\text{m}}{\text{s}^2}$
Distance between Q and A_o	L	**Specified**	$L(t) = 50 - 2\,t$ meters
$\hat{\mathbf{a}}_x$ measure of A_o's velocity in N	$\dot{x}$	**Specified**	$x(t) = 0$ meters
$\hat{\mathbf{a}}_y$ measure of A_o's velocity in N	$\dot{y}$	**Specified**	$y(t) = 0$ meters
Angle from $\hat{\mathbf{a}}_y$ to $\hat{\mathbf{b}}_y$ with $+\hat{\mathbf{a}}_z$ sense	θ	Variable	$\theta(t{=}0) = 1°$
Tension in cable	T	Variable	Not applicable

Form an equation governing Q's motion over Earth (a Newtonian reference frame N) by equating Q's resultant force to its mass and acceleration in N via $\vec{\mathbf{F}}^Q = m\,{}^N\vec{\mathbf{a}}^Q$.

Result: (Note: Homework 7.19 forms ${}^N\vec{\mathbf{a}}^Q$).

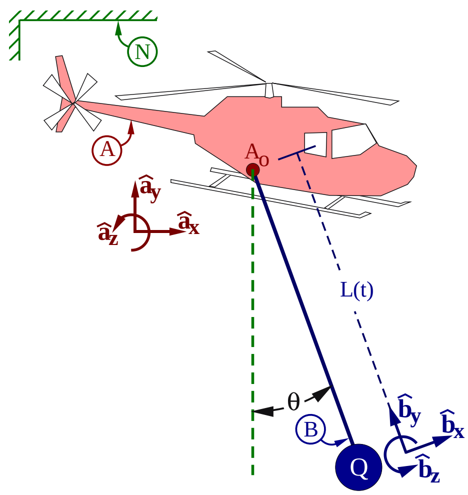
$$-m\,g\,\hat{\mathbf{a}}_y + T\,\hat{\mathbf{b}}_y \;=\; m\left[\ddot{x}\,\hat{\mathbf{a}}_x + \ddot{y}\,\hat{\mathbf{a}}_y + (L\,\ddot{\theta} + 2\,\dot{L}\,\dot{\theta})\,\hat{\mathbf{b}}_x + (-\ddot{L} + L\,\dot{\theta}^2)\,\hat{\mathbf{b}}_y\right]$$

Form a **single scalar** equation of motion in terms of $\ddot{\theta}$ and symbols in the table **except** T. Next, solve for $\ddot{\theta}$ when the helicopter is **stationary** $(\ddot{x} = \ddot{y} = 0)$.

Result:

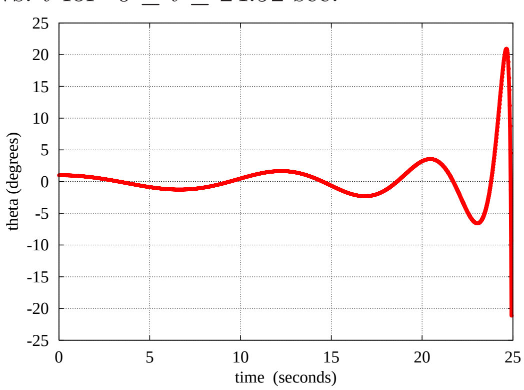
$$m\left[\cos(\theta)\,\ddot{x} + \sin(\theta)\,\ddot{y} + L\,\ddot{\theta} + 2\,\dot{L}\,\dot{\theta} + g\,\sin(\theta)\right] = 0$$

$$L\,\ddot{\theta} + 2\,\dot{L}\,\dot{\theta} + g\,\sin(\theta) = 0 \quad\Rightarrow\quad \ddot{\theta} = \frac{-2\,\dot{L}\,\dot{\theta} - g\,\sin(\theta)}{L}$$

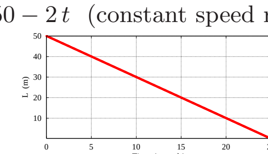


Use a computer and numerical integration step (integStp) of 0.02 sec to solve this differential equation. Use $\dot{\theta}(0) = 0\ \frac{\text{rad}}{\text{sec}}$

Plot θ vs. t for $0 \le t \le 24.92$ sec.



$L = 50 - 2\,t$ (constant speed retrieval)



Problem solution at **www.MotionGenesis.com** $\Rightarrow$ **Get Started** $\Rightarrow$ **Pendulum** and **Helicopter Retrieval**.

Notes: The high-oscillation swings may be fatal for the fishermen and helicopter and it is advantageous (albeit difficult) to specify a different function for $L(t)$, e.g., $L(t, \theta, \dot{\theta}, \ddot{\theta})$, to control this system so $\theta(t)$ stays small.

††Optional:** Find a control law for L for safe retrieval with $|\theta| \le 5°$ to $L \le 0.5$ m within 200 seconds.

Similar results are associated with sucking spaghetti into your mouth and the space-shuttle tethered satellite systems TSS1 and TSS2. TSS1 failed during deployment due to a jammed reel (the satellite was reeled back in for TSS2). TSS2 failed due to electrical voltages far in excess of those predicted by science. A pin-hole in the tether's electrical insulation allowed electrical arcing between boom and tether that broke the tether and lost the satellite. There is much unknown about Earth's electromagnetic field.