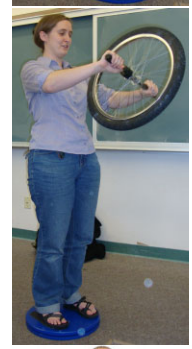
18.14 **Dynamicist on a turntable with spinning bicycle wheel (Section 22.6).**



The pictures to the right shows a dynamicist standing on a spinning turntable and holding a spinning bicycle wheel.

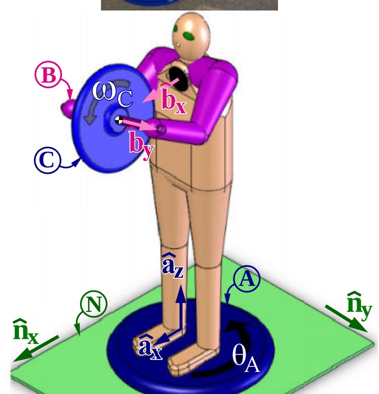
The mechanical model (below right) has a rigid body A (modeling the turntable, legs, torso, and head) that can freely rotate relative to Earth (Newtonian reference frame N) about a vertical axis that is fixed in both A and N and which passes through the center of the turntable (point N_o) and A_{cm} (A's center of mass).

A light (massless) rigid frame B (modeling the shoulders, arms, hands, and a portion of the bicycle wheel's axle) is attached to A by a revolute motor at point B_o of B (B_o lies on the vertical axis passing through A_{cm}). The motor's revolute axis passes through points B_o and C_{cm}, is horizontal, and is parallel to $\widehat{\mathbf{b}}_x = \widehat{\mathbf{a}}_x$.

A rigid bicycle wheel C is attached to B by a frictionless revolute joint whose axis passes through C_{cm} (C's center of mass) and is parallel to $\widehat{\mathbf{b}}_y$.

Right-handed orthogonal unit vectors $\widehat{\mathbf{a}}_x$, $\widehat{\mathbf{a}}_y$, $\widehat{\mathbf{a}}_z$ and $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ are fixed in A and N, respectively. Initially $\widehat{\mathbf{a}}_i = \widehat{\mathbf{n}}_i$ $(i = x, y, z)$, and then rigid body A is subjected to a right-handed rotation characterized by $\theta_A\, \widehat{\mathbf{a}}_z$ where $\widehat{\mathbf{a}}_z = \widehat{\mathbf{n}}_z$ is directed vertically-upward and $\widehat{\mathbf{a}}_x$ points from the instructors back to front (parallel to the axis of the revolute motor connecting A and B).

Unit vectors $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$ are fixed in B. Initially $\widehat{\mathbf{b}}_i = \widehat{\mathbf{a}}_i$ $(i = x, y, z)$, and then B is subjected to a right-handed rotation characterized by $\theta_B\, \widehat{\mathbf{a}}_x$ where $\widehat{\mathbf{b}}_x = \widehat{\mathbf{a}}_x$ and $\widehat{\mathbf{b}}_y$ is directed along the wheel's axle from the instructor's right hand to left hand. The dynamicist changes θ_B in a **specified** (known or prescribed) sinusoid manner with amplitude 30° and period 4 seconds.



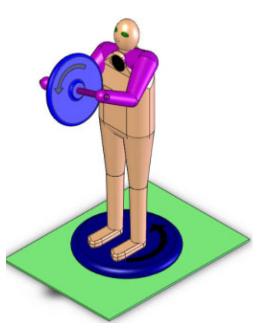
Courtesy Doug Schwandt
Purchase turntable/bicycle wheel at Arbor-scientific

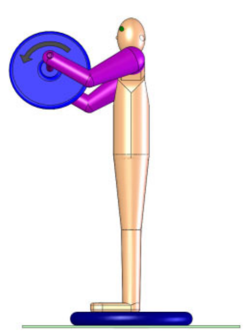
Quantity	Symbol and type		Value
Mass of C	m^C	Constant	2 kg
Distance between B_o and C_{cm}	L_x	Constant	0.5 m
A's moment of inertia about B_o for $\widehat{\mathbf{a}}_z$	I^A_{zz}	Constant	0.64 kg m²
C's moment of inertia about C_{cm} for $\widehat{\mathbf{b}}_x$	I^C	Constant	0.12 kg m²
C's moment of inertia about C_{cm} for $\widehat{\mathbf{b}}_y$	J^C	Constant	0.24 kg m²
Angle from $\widehat{\mathbf{n}}_x$ to $\widehat{\mathbf{a}}_x$ with $+\widehat{\mathbf{n}}_z$ sense	θ_A	Variable	
Angle from $\widehat{\mathbf{a}}_y$ to $\widehat{\mathbf{b}}_y$ with $+\widehat{\mathbf{a}}_x$ sense	θ_B	**Specified**	$\frac{\pi}{6}\sin(\frac{\pi}{2}t)$
$\widehat{\mathbf{b}}_y$ measure of C's angular velocity in B	ω_C	Variable	

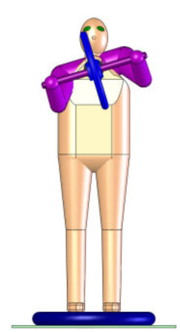
Complete the ***road map*** for finding $\ddot{\theta}_A$ and $\dot{\omega}_C$ (the "about points" are not necessarily unique).

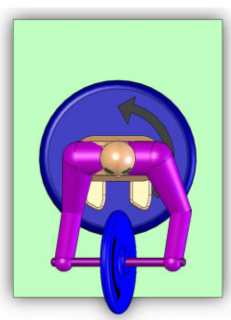
Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point*	Road-map equation
θ_A	Rotate	$\widehat{\mathbf{a}}_z$	A, B, C	**Draw**	B_o	$\widehat{\mathbf{a}}_z \cdot (\vec{\mathbf{M}}^{S/B_o} = \frac{{}^N d\, {}^N\vec{\mathbf{H}}^{S/B_o}}{dt})$
ω_C	Rotate	$\widehat{\mathbf{b}}_y$	C	**Draw**	C_{cm}	$\widehat{\mathbf{b}}_y \cdot (\vec{\mathbf{M}}^{C/C_{cm}} = \frac{{}^N d\, {}^N\vec{\mathbf{H}}^{C/C_{cm}}}{dt})$

Student/Instructor version at **www.MotionGenesis.com** ⇒ **Textbooks** ⇒ **Resources**









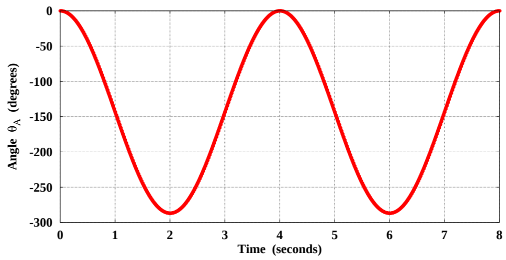
18.15 Angular momentum/conservation for dynamicist on turntable with spinning bicycle wheel.

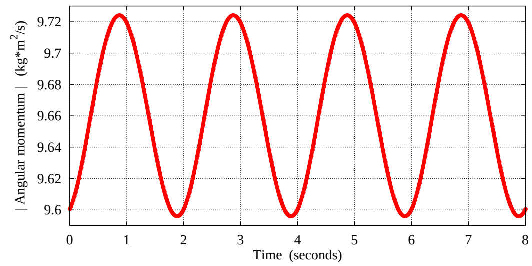
This problem refers to Homework 18.14.

(a) Using initial values of $\theta_A = 0°$, $\dot{\theta}_A = 0 \frac{\text{rad}}{\text{sec}}$, and $\omega_C = 40 \frac{\text{rad}}{\text{sec}}$, graph θ_A for $0 \le t \le 8$ sec.

Optional: Graph $\left|{}^N\vec{\text{H}}^{S/N_o}\right|$ where S consists of A, B, and C and N_o is 1.2 m below B_o.

Result:





(b) ***Conservation of angular momentum*** is frequently used to explain physical phenomena. Decide whether or not various measures of angular momentum are conserved (see Section 22.8). Explain your reasoning for each true/false answer. (Regard S as the system containing A, B, and C).

1	The magnitude of S's angular momentum about N_o in N is constant.	**True/[False]**
2	The $\hat{\mathbf{a}}_z$ measure of S's angular momentum about N_o in N is constant.	**[True]/False**
3	The magnitude of C's angular momentum about C_{cm} in N is constant.	**True/[False]**
4*	The $\hat{\mathbf{b}}_y$ measure of C's angular momentum about C_{cm} in N is constant.	**[True]/False**
5	The $\hat{\mathbf{b}}_y$ measure of C's angular velocity in N is constant.	**[True]/False**
6	The $\hat{\mathbf{b}}_y$ measure of C's angular velocity in B is constant.	**True/[False]**

Explain 1: Simulation results (above) show $\left|{}^N\vec{\text{H}}^{S/N_o}\right|$ is not constant. $\vec{\mathbf{M}}^{S/N_o} \neq \vec{\mathbf{0}}$.

Explain 2: Section 22.8 shows $\hat{\mathbf{a}}_z \cdot {}^N\vec{\text{H}}^{S/N_o}$ is constant when $\hat{\mathbf{a}}_z \cdot \vec{\mathbf{M}}^{S/N_o} = 0$ and $\hat{\mathbf{a}}_z$ is fixed in N.

Explain 3: Simulation results show $\left|{}^N\vec{\text{H}}^{C/C_{cm}}\right|$ is not constant. Also $\vec{\mathbf{M}}^{C/C_{cm}} \neq \vec{\mathbf{0}}$.

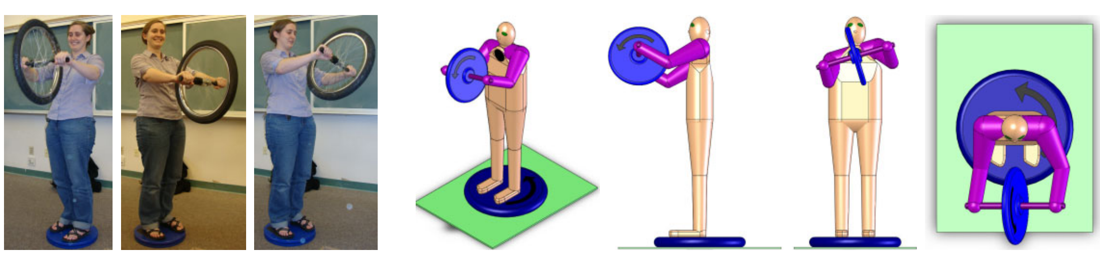
Explain 4:* (**Optional**) Referring to Section 17.2.3, C is an ***axis-symmetric*** rigid body for the line parallel to $\hat{\mathbf{b}}_y$ that passes through C_{cm} and $\hat{\mathbf{b}}_y \cdot \vec{\mathbf{M}}^{C/C_{cm}} = 0$. Alternately, q_C does not appear in $\mathcal{L}$ in *Lagrange's equations of the second kind* so $\frac{\partial \mathcal{L}}{\partial \omega_C}$ is constant, and $\hat{\mathbf{b}}_y \cdot {}^N\vec{\text{H}}^{C/C_{cm}}$ happens to be equal to $\frac{\partial \mathcal{L}}{\partial \omega_C}$.

Explain 5: Since Constant $= \hat{\mathbf{b}}_y \cdot {}^N\vec{\text{H}}^{C/C_{cm}} = J^C \left({}^N\vec{\boldsymbol{\omega}}^C \cdot \hat{\mathbf{b}}_y\right)$, then ${}^N\vec{\boldsymbol{\omega}}^C \cdot \hat{\mathbf{b}}_y =$ Constant.

Explain 6: Solution for $\dot{\omega}_C$ is non-zero, hence ω_C is not constant (verified by simulation results).

(c) A gravitational potential energy for the system S formed by A, B, C is $U_{\text{gravity}} \underset{(24.5)}{=} 0$. **[True]/False**.
Form the kinetic energy of S in N (in terms of symbols in the previous table and their time derivatives).
The sum $K + U_{\text{gravity}}$ is constant (i.e., mechanical energy is conserved). **True/[False]**. (See Section 23.4).
If not conserved, explain where work is done. Work is done by the torque on B from A.

$$K = \frac{1}{2}\text{I}^C\,\dot{\theta}_B^2 + \frac{1}{2}J^C\,\omega_C^2 + J^C\sin(\theta_B)\,\dot{\theta}_A\,\omega_C + \frac{1}{2}\left[\text{I}^A_{zz} + m^C L_x^2 + \text{I}^C\cos(\theta_B)^2 + J^C\sin(\theta_B)^2\right]\dot{\theta}_A^2$$



Solution at www.MotionGenesis.com ⇒ Get Started ⇒ Angular momentum/gyros.