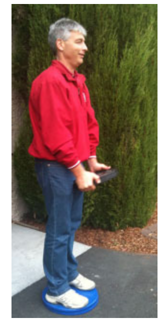
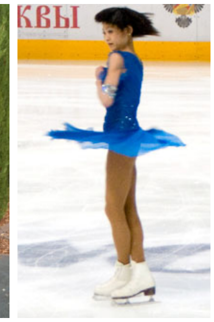
22.6.5 MG road-map: Dynamicist on a turntable (ice-skater)





A dynamics instructor stands on a spinning turntable and swings a heavy dumbbell inward and outward to change his spin-rate (similar to the ice-skater).

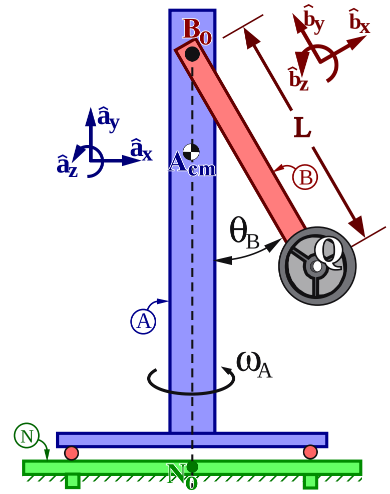
The schematic (below-right) shows a rigid body A (modeling the instructor's legs, torso, and head) that rotates (without friction) relative to Earth (a Newtonian reference frame N) about a vertical axis that is fixed in both A and N and which passes through both point N_o of N and point A_{cm} (A's center of mass).

A massless rigid arm B (modeling the instructor's arms and hands) attaches to A by a revolute motor (shoulder/muscles) whose revolute axis is horizontal and located at point B_o of B (B_o lies on the vertical axis connecting N_o and A_{cm}).

The motor (muscles) **specifies** B's angle θ_B relative to A to change in a known (prescribed) manner from 0 to π rad in 4 seconds $(\theta_B = \pi \, \frac{t}{4})$.

A heavy dumbbell Q (modeled as a particle) is rigidly attached (welded) to the end of B (the instructor's hands).

Right-handed orthogonal unit vectors $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ and $\hat{\mathbf{b}}_x$, $\hat{\mathbf{b}}_y$, $\hat{\mathbf{b}}_z$ are fixed in A and B, respectively, with $\hat{\mathbf{a}}_y$ vertically-upward, $\hat{\mathbf{b}}_z = \hat{\mathbf{a}}_z$ parallel to the revolute motor's axis, and $\hat{\mathbf{b}}_y$ directed from Q to B_o.



Quantity	Symbol	Type	Value
Earth's gravitational constant	g	Constant	$9.8 \frac{\text{m}}{\text{s}^2}$
Distance between Q and B_o	L	Constant	0.7 m
Mass of Q	m	Constant	12 kg
A's moment of inertia about line $\overline{A_{cm} B_o}$	I_{yy}	Constant	0.6 kg m^2
Angle from $\hat{\mathbf{a}}_y$ to $\hat{\mathbf{b}}_y$ with $+\hat{\mathbf{a}}_z$ sense	θ_B	**Specified**	$\mathbf{0.25 \, \pi \, t}$ rad
$\hat{\mathbf{a}}_y$ measure of A's angular velocity in N	ω_A	Variable	

Complete the ***MG road-map*** for the turntable's "spin-rate" ω_A (Note: The "about point" is not unique)

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point*	***MG road-map equation***
ω_A	Rotate	$\hat{\mathbf{a}}_y$	A, B, Q	**Draw**	B_o	$\hat{\mathbf{a}}_y \cdot (\vec{\mathbf{M}}^{S/B_o} = \frac{^N d \; ^N\vec{\mathbf{H}}^{S/B_o}}{dt})$
ω_A	**Dot(Ay>, System(A, B, Q).GetDynamics(Bo))**					**M**otion**G**enesis command ©

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