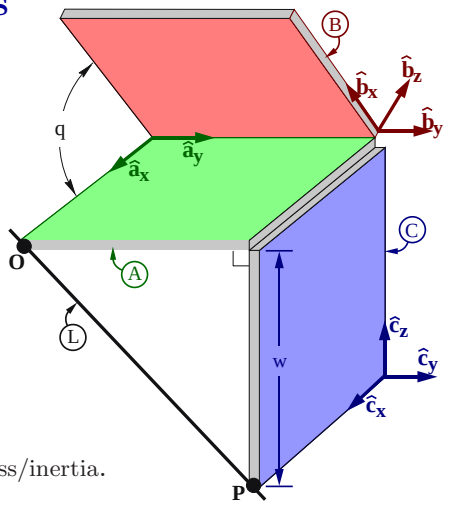
7.3 Mass, mass center, and inertia calculations



The figure to the right shows three identical uniform hinge-connected plates A, B, and C.

Right-handed sets of mutually perpendicular unit vectors $\hat{\mathbf{a}}_i$, $\hat{\mathbf{b}}_i$, and $\hat{\mathbf{c}}_i$ $(i = x, y, z)$ are fixed in A, B, and C, respectively, with $\hat{\mathbf{a}}_y = \hat{\mathbf{b}}_y = \hat{\mathbf{c}}_y$ parallel to the hinge connecting A and B.

The plates are thin and square and have dimension $w = 1$ meter and mass $m = 12$ kg.

Points O and P mark corners of A and C and the angle q characterizes B's orientation in A.

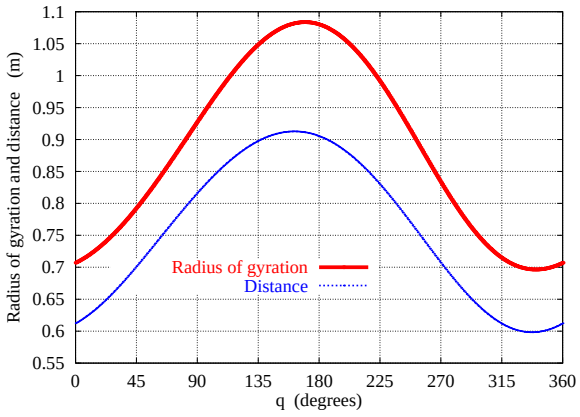
Note: Problem solution at www.MotionGenesis.com ⇒ Get Started ⇒ Plate mass/inertia.

1. Find the distance between line $\overline{OP}$ and the center of mass S_{cm} of the system formed by A, B, C.
   Result:
   $$\text{Distance} = 0.1178511\sqrt{42 + \cos^2(q) + 6\sin(q) - 16\cos(q)}$$

2. For $q = 90°$, find λ_i $(i=1,2,3)$, the system's principal moments of inertia about S_{cm}. Next, find the angle between $\hat{\mathbf{a}}_y$ and the principal axis associated with this system's **minimum** moment of inertia.
   Result: $\lambda_1 = 5\ \text{kg m}^2$ $\lambda_2 = 13\ \text{kg m}^2$ $\lambda_3 = 14\ \text{kg m}^2$ Angle $= 65.90516°$

3. The system's radius of gyration about line $\overline{OP}$ is a measure of the how far the mass distribution is from line $\overline{OP}$ and is defined as $\sqrt{I/m}$ where I is the system's moment of inertia about $\overline{OP}$ and m is the system's mass. Using **physical intuition**, estimate the values of q that produce the smallest and largest radii of gyration about line $\overline{OP}$ and provide a reason for choosing these values.
   Result: $q_{small} \approx$ 340° $q_{large} \approx$ 160° ($0 \le q \le 360°$)
   Reason: These values minimize or maximize the distance from line OP to B's mass center.

4. Plot the system's mass center distance from line $\overline{OP}$ and the system's radius of gyration about line $\overline{OP}$ for $0 \le q \le 360°$. Determine the minimum/maximum distance and radius of gyration and associated values of q.



	Minimum Value	q	Maximum Value	q
Distance	0.598 m	337°	0.913 m	161°
Gyration	0.696 m	340°	1.084 m	169°