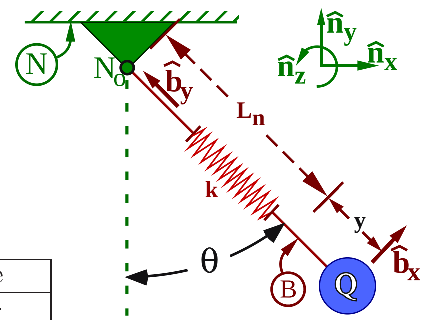
20.5 Kane/Lagrange methods for a swinging-spring (refer to Hw 8.14).



A straight massless spring connects a particle Q to a point N_o (fixed in a Newtonian reference frame N).

Right-handed orthogonal unit vectors $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ are fixed in N with $\hat{\mathbf{n}}_x$ horizontally-right and $\hat{\mathbf{n}}_y$ vertically-upward.

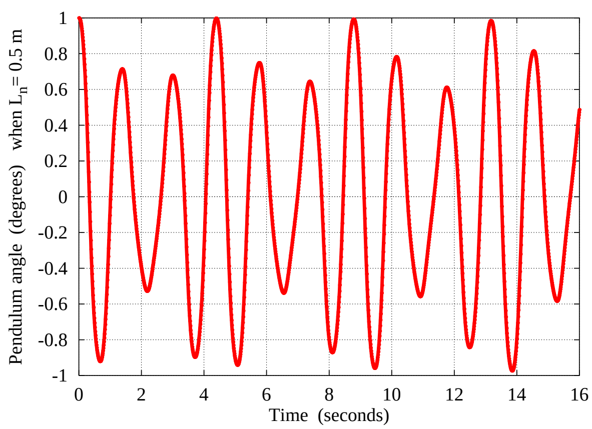
Right-handed orthogonal unit vectors $\hat{\mathbf{b}}_x$, $\hat{\mathbf{b}}_y$, $\hat{\mathbf{b}}_z$ are fixed in a reference frame B with $\hat{\mathbf{b}}_y$ directed from Q to N_o and $\hat{\mathbf{b}}_z = \hat{\mathbf{n}}_z$.

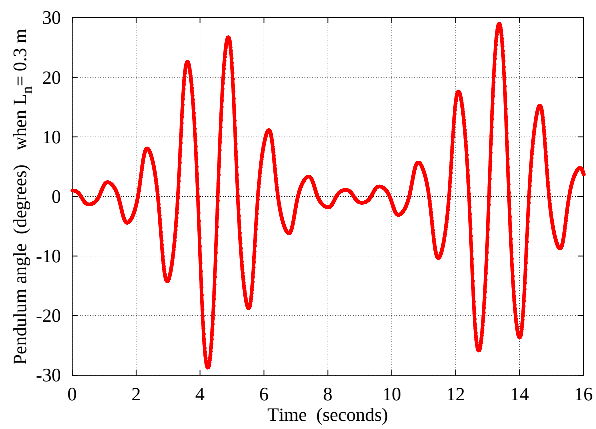
Quantity	Symbol	Type	Value
Earth's gravitational constant	g	Constant	$9.8 \frac{m}{s^2}$
Mass of Q	m	Constant	1 kg
Natural spring length	L_n	Constant	0.5 m
Linear spring constant	k	Constant	$100 \frac{N}{m}$
Spring stretch	y	Variable	$y(0) = 0.2$ m
Angle from $\hat{\mathbf{n}}_y$ to $\hat{\mathbf{b}}_y$ with $+\hat{\mathbf{n}}_z$ sense	θ	Variable	$\theta(0) = 1°$

- Form ***Kane's equations*** for ***generalized speeds*** $\dot{x}$ and $\dot{\theta}$ and/or ***Lagrange's equations*** for ***generalized coordinates*** x and θ and/or ***MG road-maps*** for x and θ. Solve for $\ddot{x}$ and $\ddot{\theta}$.

Result:
$$\ddot{y} = g \cos(\theta) + (L_n + y)\,\dot{\theta}^2 - \frac{k\,y}{m} \qquad \ddot{\theta} = \frac{-[g \sin(\theta) + 2\,\dot{\theta}\,\dot{y}]}{L_n + y}$$

- Using the values in the previous table and $\dot{y}(0) = \dot{\theta}(0) = 0$, simulate the motion of this swinging spring for $0 \le t \le 16$ sec. Plot θ vs. t. Plot again with $L_n = 0.3$ m.





- Comparing the two simulations, the magnitude of θ is significantly larger when $L_n =$ 0.3 m.
- The simulation results for this "***simple planar problem***" are intuitive and obvious. **True/False.**

Note: More information on the swinging spring is in [13, pgs. 62, 129-137], [?], [?]. A physical demonstration of this system is afforded with a fishing line, spring, and mass. After experimentally determining m and k, choose a length of line so the equilibrium length (i.e., the natural length and stretch due to gravity) is $\approx \frac{4\,m\,g}{k}$. Lastly, pull Q (the mass) straight down and release it. Although Q initially moves mostly vertical, it transitions to a swinging motion. After reeling in (or out) more fishing line, the coupled motion (i.e., the nonlinear beat phenomenon) is no longer visible. Solution (including Kane/Lagrange) and simulation at **www.MotionGenesis.com** ⇒ **Get Started** ⇒ **Pendulum**.