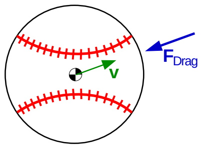
11.10 Baseball trajectories with air-resistance. $\boxed{\vec{\mathbf{F}} \;\Rightarrow\; \vec{\mathbf{F}} = m\,\vec{\mathbf{a}} \;\Rightarrow\; \ddot{x},\, \ddot{y} \;\Rightarrow\; x,\, y}$

The baseball trajectories in "*The Physics of Baseball*" by Robert Adair[a] and "*How to hit home runs ...*" by Sawicki, Hubbard, Stronge[b] are ***significantly different***. For a certain condition, Sawicki, Hubbard, Stronge [71, p. 1159] predict travel of **442 ft** (≈ 135 m) whereas Adair's [2] predicts a maximum travel of **351 ft** (≈ 107 m). They disagree on the relationship between ***coefficient of drag*** C_D and $\vec{\mathbf{v}}$ in a standard aerodynamic drag, which in still air is



$$\vec{\mathbf{F}}_{\text{Drag}} \;=\; -\tfrac{1}{2}\,\rho\,A\,C_D\,\left|\vec{\mathbf{v}}\right|\,\vec{\mathbf{v}}$$

$\vec{\mathbf{v}}$ is the velocity of the baseball's centroid
ρ is the density of air ($0.075\,\frac{\text{lbm}}{\text{ft}^3} \approx 1.2\,\frac{\text{kg}}{\text{m}^3}$ at sea level)

- $A = \pi\, r^2$ is the ball's cross-sectional area normal to $\vec{\mathbf{v}}$.
- $r = 1.44$ inches (≈ 3.66 cm) is the baseball's radius.

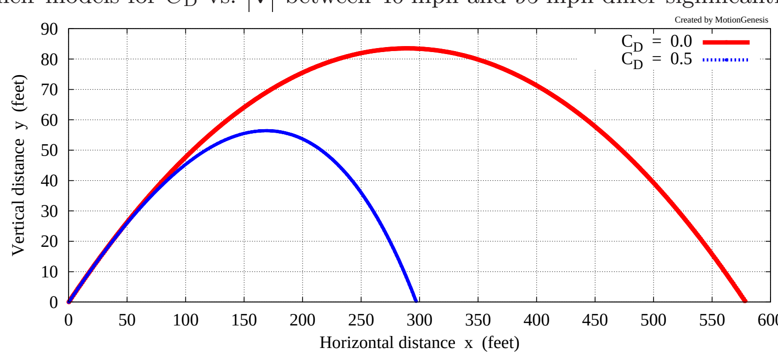
[a] Yale physics professor Robert Adair is a member of the National Academy of Sciences.
[b] Sawicki, Hubbard, and Stronge are professors at Michigan (Ann Arbor), California (Davis), and Cambridge.

Using Earth's gravity $g = 32.2\,\frac{\text{ft}}{\text{s}^2}$ ($\approx 9.8\,\frac{\text{m}}{\text{s}^2}$), a regulation baseball of mass $m = 5$ ozm (≈ 142 grams), a **constant** ***drag coefficient*** $C_D = 0.5$, and neglecting other aerodynamic forces, determine the baseball's ***terminal velocity***. Note: A terminal velocity of 95 mph was experimentally determined in wind-tunnel tests by Briggs. Adair and Sawicki, Hubbard, Stronge both agree that at low-speeds (≤ 40 mph) $C_D = 0.5$, whereas at 95 mph $C_D \approx 0.32$. However, their models for C_D vs. $\left|\vec{\mathbf{v}}\right|$ between 40 mph and 95 mph differ significantly.

Result:

terminal velocity $=$ $\boxed{74}$ mph
Error in prediction $=$ $\boxed{-22}$ %

Create "x-y" plots of the baseball trajectories when hit from home plate ($x = 0$, $y = 0$) at 100 mph ($44.7\,\frac{\text{m}}{\text{s}}$) with a launch angle $\theta = 30°$ with the horizontal.
Result shown right.



Using $C_D = 0.5$, estimate x_{hit} (the value of x when the baseball lands on a flat horizontal playing field).
Using $C_D = 0$, plot the baseball's trajectory (on the previous plot) and again estimate x_{hit}.
Print and submit your simulation plots and programs (e.g., **M**otion**G**enesis or MATLAB®). **Results below.**

Optional: For each value of C_D, determine whether the system has a potential energy, whether the gravitational forces on the baseball have a potential energy, and whether the drag forces on the baseball have a potential energy. Determine whether the work done by the drag forces on the baseball from $t = 0$ to the value of t at which the ball lands is negative ($-$), zero (0), or positive ($+$).

Optional: Estimate the optimal launch angle (for this drag model) to maximize x_{hit} (to within 1°).

C_D	θ_{launch}	x_{hit}	System has a potential energy	Gravity has a potential	Drag has a potential	Work done by gravity	Work done by drag	θ_{optimal}	x_{optimal}
0.5	30°	297 ft	**True/False**	**True/False**	**True/False**	$-$ / 0 / +	$-$ / 0 / +	39°	309 ft
0	30°	579 ft	**True/False**	**True/False**	**True/False**	$-$ / 0 / +	$-$ / 0 / +	45°	668 ft

Solution at **www.MotionGenesis.com** $\Rightarrow$ **Get Started** $\Rightarrow$ **Projectile motion**.

Air-resistance impacts a baseball's trajectory, **halving** its maximum distance. Moreover, both Adair and SHS assert C_D varies with $\vec{\mathbf{v}}$. With lift, drag, spin, etc., Adair's optimal launch angle is $\approx 35°$ whereas SHS's is $\approx 26°$.