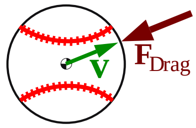
FE/EIT Review – Baseball trajectory.

$\boxed{\vec{\mathbf{F}} = m\,\vec{\mathbf{a}} \;\Rightarrow\; \ddot{x},\ \ddot{y} \;\Rightarrow\; x,\ y}$ (model in Section 19.15)

The distance a baseball travels of **351 ft** (≈ 107 m) predicted by Robert Adair[a] in "*The Physics of Baseball*" [2], **differs significantly** from the **442 ft** (≈ 135 m) predicted by Sawicki, Hubbard, Stronge[b] in "*How to hit home runs ...*" [53, p. 1159].

Although they use the same aerodynamic drag model for $\vec{\mathbf{F}}_{\text{Drag}}$, they disagree on the relationship between ***coefficient of drag*** C_{D} and $|\vec{\mathbf{v}}|$ (baseball speed).



$$\vec{\mathbf{F}}_{\text{Drag}} = -\tfrac{1}{2}\,\rho\,A\,C_{\text{D}}\,|\vec{\mathbf{v}}|^2\,\hat{\mathbf{u}} = -\tfrac{1}{2}\,\rho\,A\,C_{\text{D}}\,|\vec{\mathbf{v}}|\;\vec{\mathbf{v}}$$

$\vec{\mathbf{v}}$ is the velocity of the baseball's centroid and $\hat{\mathbf{u}} = \frac{\vec{\mathbf{v}}}{|\vec{\mathbf{v}}|}$.

ρ is the density of air (at sea level, $\rho \approx 0.075\ \frac{\text{lbm}}{\text{ft}^3} \approx 1.2\ \frac{\text{kg}}{\text{m}^3}$).

$A = \pi\,r^2$ is the baseball's cross-sectional area and $r = 1.44$ inches ≈ 3.66 cm is its radius.

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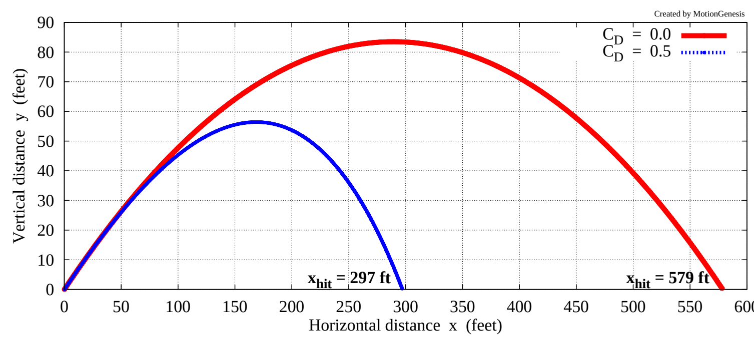
[a] Yale physics professor Robert Adair is a member of the National Academy of Sciences.
[b] Sawicki, Hubbard, and Stronge are professors at Michigan (Ann Arbor), California (Davis), and Cambridge.

Using gravity $g = 32.2\ \frac{\text{ft}}{\text{s}^2}$ ($\approx 9.8\ \frac{\text{m}}{\text{s}^2}$), a regulation baseball of mass $m = 5$ ozm (≈ 142 grams), $C_{\text{D}} = 0.5$ (**constant**), and neglecting other aerodynamic forces, determine the baseball's ***terminal velocity***. Using $C_{\text{D}} = 0$, calculate x_{hit} (the value of x when the baseball lands on a flat playing field).

Result: (hint for x_{hit}, see Section 8.3)
terminal velocity $\approx$ $\boxed{74}$ mph
When $C_{\text{D}} = 0$, $x_{\text{hit}} \approx \boxed{579}$ ft

Optional: Create an "x-y" plot of the baseball's trajectory when it is hit from home plate ($x = 0$, $y = 0$) at 100 mph ($44.7\ \frac{\text{m}}{\text{s}}$) with a launch angle $\theta = 30°$ above the horizontal. **Result shown right.**



Optional: Show $x_{\text{hit}} \approx \boxed{297}$ ft when $C_{\text{D}} = 0.5$ and plot the baseball's trajectory. **Results above.** Print and submit your simulation plots and programs (e.g., **M**otion**G**enesis or MATLAB®).

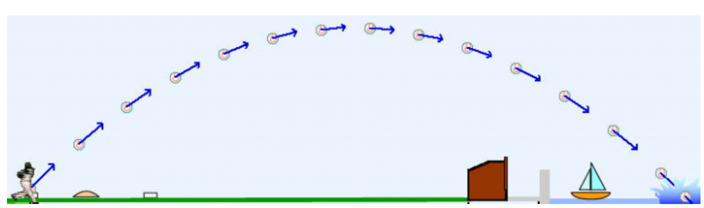
Optional: For each value of C_{D}, determine whether the system has a potential energy, whether the gravitational forces on the baseball have a potential energy, and whether the drag forces on the baseball have a potential energy. Determine whether the work done by the drag forces on the baseball from $t=0$ to the value of t at which the ball lands is negative (–), zero (0), or positive (+).

Optional: Estimate the optimal launch angle (for this drag model) to maximize x_{hit} (to within 1°).

C_{D}	θ_{launch}	x_{hit}	System has a potential energy	Gravity has a potential	Drag has a potential	Work done by gravity	Work done by drag	θ_{optimal}	x_{optimal}
0.5	30°	$\boxed{297}$ ft	True/$\boxed{\text{False}}$	$\boxed{\text{True}}$/False	True/$\boxed{\text{False}}$	– / $\boxed{0}$ / +	$\boxed{–}$ / 0 / +	$\boxed{39}$°	$\boxed{309}$ ft
0	30°	$\boxed{579}$ ft	$\boxed{\text{True}}$/False	$\boxed{\text{True}}$/False	$\boxed{\text{True}}$/False	– / $\boxed{0}$ / +	– / $\boxed{0}$ / +	$\boxed{45}$°	$\boxed{668}$ ft

Solution at **www.MotionGenesis.com** ⇒ **Get Started** ⇒ **Projectile motion**.

Note: Air-resistance impacts a baseball's trajectory, **halving** its maximum distance. Both Adair and SHS assert C_{D} varies with $\vec{\mathbf{v}}$. With lift, drag, spin, etc., Adair's optimal launch angle is $\approx 35°$ whereas SHS's is $\approx 26°$.



Note: A terminal velocity of 95 mph was experimentally determined in wind-tunnel tests by Briggs. Adair and Sawicki, Hubbard, Stronge both agree that at low-speeds (≤ 40 mph) $C_{\text{D}} = 0.5$ and at 95 mph $C_{\text{D}} \approx 0.32$. However, their models for C_{D} vs. $|\vec{\mathbf{v}}|$ between 40 mph and 95 mph **differ significantly**.