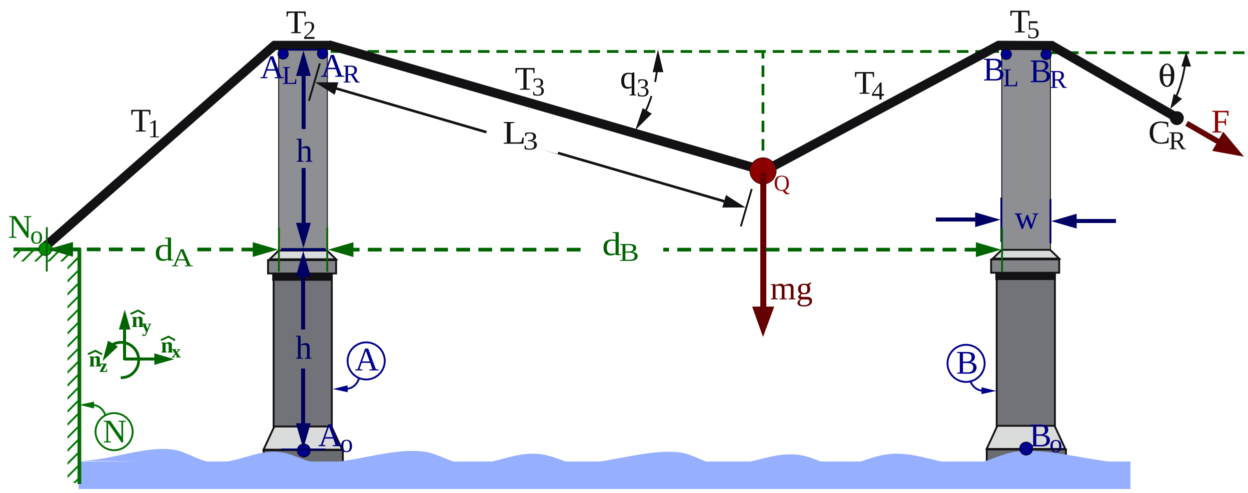
5.9 Static equilibrium of a suspension bridge with pulleys.

The following figure depicts a construction phase of a suspension bridge. A relatively thin, light (**massless**), inextensible cable is firmly attached to ground and is pass through light, frictionless, pulleys at the tops of two identical support towers. Massive objects supported by the cable are modeled as a particle Q ***welded*** to the cable.

The cable C is connected to point N_o of ground N, then rises to the right until it encounters a pulley attached to tower A at point A_L (the top-left-most point of A). The cable is horizontal over the top of tower A until it encounters a pulley attached to tower A at point A_R (the top-right-most point of A). The cable inclines down to the right until it meets particle Q and then rises to the right until it encounters a pulley attached to tower B at point B_L (the top-left-most point of B). The cable is horizontal over the top of tower B until it encounters a pulley attached to tower B at point B_R (the top-right-most point of B). After B_R, the cable inclines down to the right by an angle θ due to a force applied to point C_R (the right-most end of C).



$\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ are right-handed orthogonal unit vectors with $\widehat{\mathbf{n}}_x$ horizontally-right and $\widehat{\mathbf{n}}_y$ vertically-upward.

Generate **one** equation relating q_3 to F. Complete the table with numerical values for q_3 and T_i ($i = 1, ..., 5$).

Result:

$$m\,g\,\cos(q_3) \;-\; \frac{d_B}{L_4}\,F\,\sin(q_3) \;=\; 0$$

Quantity	Symbol	Value
Mass of particle Q	m	$100,000$ kg
Earth's gravitational acceleration	g	9.8 m/s^2
Magnitude of force applied to C_R	F	$800,000$ N
Angle between $\widehat{\mathbf{n}}_x$ and line $\overline{B_R\,C_R}$	θ	$20°$
Distance between N_o and tower A	d_A	40 m
Width of towers (horizontal)	w	5 m
Distance between towers A and B	d_B	100 m
Height of towers above N_o (vertical)	h	30 m
Distance between point A_R and Q	L_3	80 m
Angle between $\widehat{\mathbf{n}}_x$ and line $\overline{A_R\,Q}$	q_3	$35.3°$
Tension in cable between N_o and A_L	T_1	$588,626$ N
Tension in cable between A_L and A_R	T_2	$588,626$ N
Tension in cable between A_R and Q	T_3	$588,626$ N
Tension in cable between Q and B_L	T_4	$800,000$ N
Tension in cable between B_L and B_R	T_5	$800,000$ N

5.10 Optional**: Static equilibrium of a suspension bridge without pulleys.

Repeat the previous analysis except consider the cable as pulled over the smooth (**frictionless**) tops of the two support towers (pulleys are **not** used).

Note: There are various ways to solve this problem and its nonlinear algebraic equations.

Quantity	Symbol	Value
Angle between $\hat{\mathbf{n}}_x$ and line $\overline{A_R\,Q}$	q_3	17.04°
Tension in cable between N_o and A_L	T_1	939,693 N
Tension in cable between A_L and A_R	T_2	751,754 N
Tension in cable between A_R and Q	T_3	786,275 N
Tension in cable between Q and B_L	T_4	1,061,604 N
Tension in cable between B_L and B_R	T_5	751,754 N

Verify the calculations to the right (useful for this analysis).

Complete the table below.

Description	Equation
Distance between N_o and A_L	$L_1 = \sqrt{d_A^2 + h^2}$
Distance between B_L and Q	$L_4 = \sqrt{L_3^2 \sin^2(q_3) + [d_B - L_3 \cos(q_3)]^2}$
N_o's unit position vector from A_L	$\hat{\mathbf{u}}^{N_o/A_L} = \frac{-d_A}{L_1}\,\hat{\mathbf{n}}_x - \frac{h}{L_1}\,\hat{\mathbf{n}}_y$
Q's unit position vector from A_R	$\hat{\mathbf{u}}^{Q/A_R} = \cos(q_3)\,\hat{\mathbf{n}}_x - \sin(q_3)\,\hat{\mathbf{n}}_y$
Q's unit position vector from B_L	$\hat{\mathbf{u}}^{Q/A_R} = \frac{d_B - L_3 \cos(q_3)}{L_4}\,\hat{\mathbf{n}}_x - \frac{L_3 \sin(q_3)}{L_4}\,\hat{\mathbf{n}}_y$
C_R's unit position vector from B_R	$\hat{\mathbf{u}}^{C_R/B_R} = \cos(\theta)\,\hat{\mathbf{n}}_x - \sin(\theta)\,\hat{\mathbf{n}}_y$

Quantity	Tower A	Tower B
Cable resultant force on tower:	$\vec{\mathbf{F}}^{A/\text{cable}} =$ ______ $\hat{\mathbf{n}}_y$ Newtons	$\vec{\mathbf{F}}^{B/\text{cable}} =$ ______ $\hat{\mathbf{n}}_y$ Newtons
Cable forces on tower are:	**Compressive/Tensile**	**Compressive/Tensile**
$\lvert\vec{\mathbf{F}}^{Tower/\text{cable}}\rvert < m\,g + F\,\sin(\theta)$	**True/False**	**True/False**
Moment of forces from cable on tower:	$\vec{\mathbf{M}}^{A/A_o} =$ ______ $\hat{\mathbf{n}}_z$	$\vec{\mathbf{M}}^{B/B_o} =$ ______
Cable forces cause tower to bend:	**Clockwise/Counter-clockwise**	**Clockwise/Counter-clockwise**

Problem solution at www.MotionGenesis.com ⇒ Get Started ⇒ Suspension bridge.

- **Optional**:** Draw B's bending moment diagram as a function of a vertical measure y.
- **†Optional**:** Re-analyze with a coefficient of friction between cable and tower of $\mu_s = 0.2$.