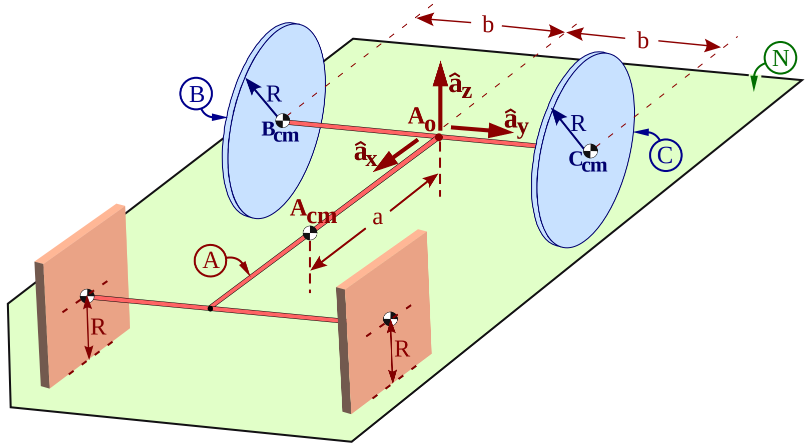
22.15 Investigation of vehicle skidding (refers to Hw 10.6).

The figure below is a schematic representation of a vehicle S that consists of a rigid chassis A and two identical uniform rigid wheels, B and C that ***roll*** on a horizontal plane N. Wheels B and C are free to rotate relative to A on an axle whose midpoint is A_o. This vehicle's other two wheels are "locked up" and are modeled as rigidly attached to A and sliding ***without friction*** on N.

Right-handed orthogonal unit vectors $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ are fixed in A with $\hat{\mathbf{a}}_z$ perpendicular to N and $\hat{\mathbf{a}}_y$ parallel to A's front and rear axles.



Description	Symbol	Type	(initial) value
Radius of wheels B and C	R	Constant	0.35 m
Distance from center of each wheel to point A_o	b	Constant	0.75 m
Distance from A_o to to A_{cm} (A's center of mass)	a	Constant	1.64 m
Mass of A	m^A	Constant	640 kg
Mass of B or C	m	Constant	30 kg
A's moment of inertia about A_{cm} for $\hat{\mathbf{a}}_z$	I^A	Constant	166.6 kg m^2
B's moment of inertia about B_{cm} for $\hat{\mathbf{a}}_x$ or $\hat{\mathbf{a}}_z$	J	Constant	2 kg m^2
B's moment of inertia about B_{cm} for $\hat{\mathbf{a}}_y$ (same for C)	K	Constant	1 kg m^2
$\hat{\mathbf{a}}_x$ measure of A_o's velocity in N $({}^N\vec{\mathbf{v}}^{A_o} = v\,\hat{\mathbf{a}}_x)$	v	Variable	(± 25 m/s)
$\hat{\mathbf{a}}_z$ measure of A's angular velocity in N $({}^N\vec{\boldsymbol{\omega}}^A = w\,\hat{\mathbf{a}}_z)$	w	Variable	(0.01 rad/sec)

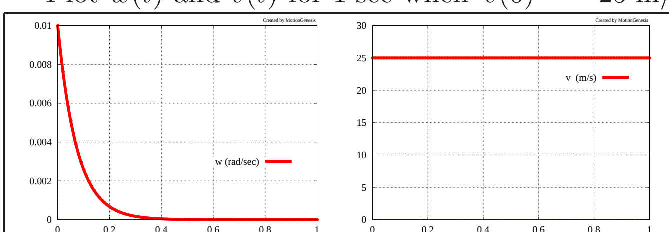
Rolling constraints from Hw 10.6:

$$\omega_B = \frac{v + b\,w}{R}$$

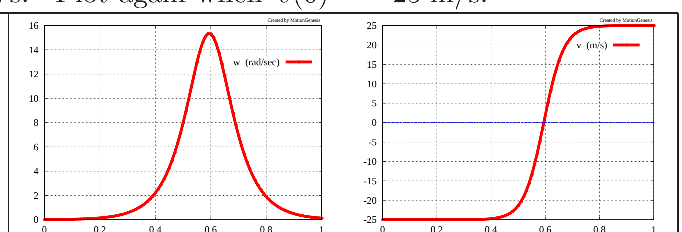
$$\omega_C = \frac{v - b\,w}{R}$$

Reminder: To enforce B's rolling on N, there must be a static friction force at B's contact point with N. Similarly for C's rolling on N.

Plot $w(t)$ and $v(t)$ for 1 sec when $v(0) = {}^+25$ m/s. Plot again when $v(0) = -25$ m/s.



$\mathbf{w(t)}$ and $\mathbf{v(t)}$ when $v(0) = {}^+25\ \frac{\text{m}}{\text{s}}$

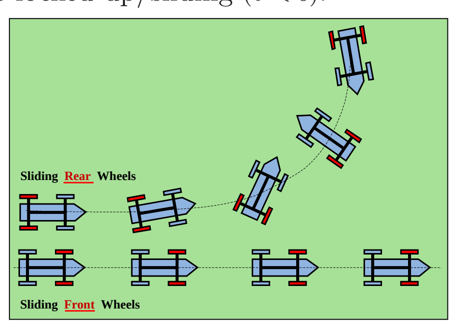


$\mathbf{w(t)}$ and $\mathbf{v(t)}$ when $v(0) = -25\ \frac{\text{m}}{\text{s}}$

These results help predict the behavior of a car with its back wheels rolling and front wheels locked-up/sliding $(v > 0)$ or with its front wheels rolling and back wheels locked-up/sliding $(v < 0)$.

When the **[front]/back** (circle one) wheels skid, $w(t)$ decays exponentially (seems stable), whereas when the **front/[back]** wheels skid, $w(t)$ grows exponentially (seems unstable).

This analysis directly relates to skidding-vehicle schematic shown right which matches 1 second of motion simulation (numerically integrating the equations of motion).



Solution at **www.MotionGenesis.com** ⇒ **Get Started** ⇒ **Vehicle Skid.**

(a) Draw a ***free-body diagram (FBD)*** of rolling wheel B. Explain why no force or torque in that FBD contributes to the power or generalized forces for the system S consisting of A, B, C.

(b) Calculate ***generalized effective forces*** ${}^N\mathcal{F}_r^S = {}^N\mathcal{F}_r^A + {}^N\mathcal{F}_r^B + {}^N\mathcal{F}_r^C$ for $u_r = w$ and $u_r = v$

and set them to zero (since ***generalized forces are zero***). Show the ODEs governing w and v are

where z_1 and z_2 are positive constants

$$\dot{w} = -z_1\, w\, v \qquad \dot{v} = z_2\, w^2$$

$$z_1 = \frac{m^A\, a}{2Jb^2/R^2 + \mathrm{I}^A + 2\,K + m^A\, a^2 + 2\,m\,b^2} \qquad z_2 = \frac{m^A\, a}{2J/R^2 + m^A + 2\,m}$$

(c) Form dynamic equations with the "clever" ***MG road-map/D'Alembert's method***.
Pick systems that eliminate all but two constraint force measures (e.g., $F_x^{B_N}$ and $F_x^{C_N}$).
For maximum simplification, first solve the 2 MG road-maps that correspond to ω_B, ω_C for $F_x^{B_N}$, $F_x^{C_N}$. Then substitute these expressions into the MG road-maps for v and w.
Result: $F_x^{B_N} = \frac{-J}{R}\,\dot{\omega}_B = \frac{-J}{R^2}\,(\dot{v} + b\,\dot{w})$ $\qquad F_x^{C_N} = \frac{-J}{R}\,\dot{\omega}_C = \frac{-J}{R^2}\,(\dot{v} - b\,\dot{w})$

(d) **Optional:** Draw individual ***free-body diagrams (FBDs)*** of A, B, C. Form the Newton/Euler equations for each individual rigid body. †Then simplify to the previous results.

This analysis gives insights into a variety of interesting facts.

• There are two brake-lines on cars because if one punctures, the other can still stop the vehicle. Normally, there is one brake-line for the front-right and rear-left wheels and another for the front-left and rear-right wheels. Although generally unwise to build a car with one brake-line for the front wheels and another for the rear wheels because a punctured front brake-line would result in unstable braking (the figure to the right shows the car moving into another lane of traffic and spinning out of control), this design can be useful for dramatic car-chase scenes in movies.

• This phenomenon is one reason modern aircraft have **one** nose-wheel and **two** tail-wheels. The old DC-3 had two nose-wheels and one tail-wheel and frequently did ground-loops (unstable).

• This analysis gives loosely-related insight into why it is easier to push (rather than pull) a tricycle, shopping cart, or golf cart.

Note: When $v(0) > 0$ (front wheels sliding, back wheels rolling), MotionGenesis's linearization and stability analysis predict that perturbations decay exponentially whereas when $v(0) < 0$ (front wheels rolling, back wheels sliding), the linearized ODEs and stability analysis predict that perturbations grow exponentially.

Solution at www.MotionGenesis.com ⇒ Get Started ⇒ **Vehicle Skid.**