10.22 Tether-ball: Simulate particle wrapping around cylinder (it is "just geometry").

The following figures show a particle Q of mass $m = 100$ g attached to a thin massless taut inextensible (constant-length) string B. The other end of the string is wrapped around a horizontal cylinder N of radius $R = 3$ cm that is rigidly fixed on Earth (a Newtonian reference frame). The taut string is tangent to the cylinder's surface.

Right-handed unit vectors $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ are fixed in N with $\hat{\mathbf{n}}_z$ along the cylinder's axis. Point N_o is fixed on the cylinder along its symmetry axis. Initially, Q's position from N_o is $R\,\hat{\mathbf{n}}_x + (70\text{ cm})\,\hat{\mathbf{n}}_y$, and Q's initial velocity is $(v_0 = -4\ \frac{\text{m}}{\text{s}})\,\hat{\mathbf{n}}_x$.

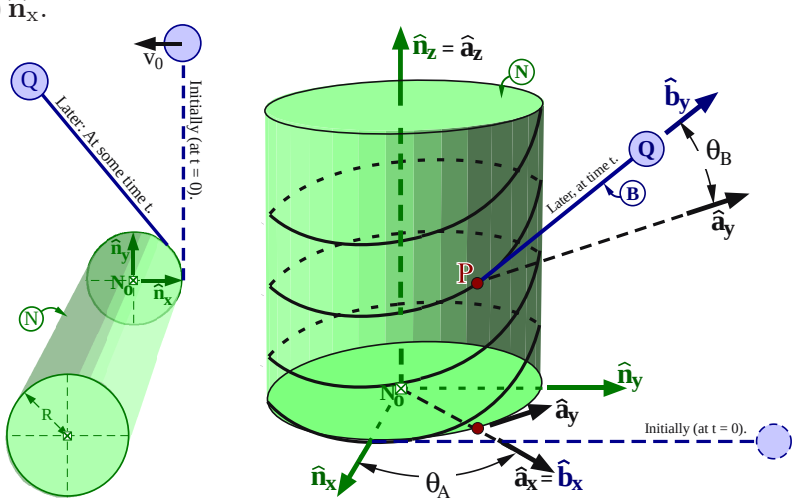
The horizontal cylinder (left) has $\hat{\mathbf{n}}_y$ vertical and $\hat{\mathbf{n}}_z$ horizontal.

The vertical cylinder (right) has $\hat{\mathbf{n}}_y$ horizontal and $\hat{\mathbf{n}}_z$ vertical.

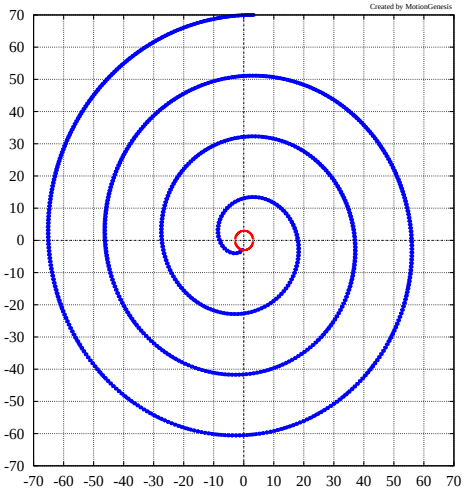
Answer each question below and plot Q's trajectory in N until the instant before Q hits the cylinder.

Use Earth's gravitational constant $g = 0$ or $g = 9.8\ \frac{\text{m}}{\text{s}^2}$.

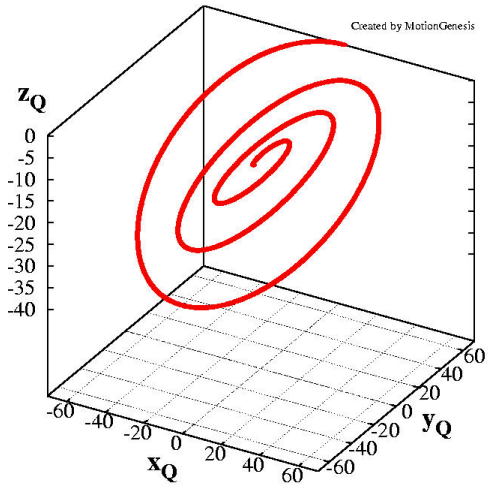
Note: Problem relates to tether-ball and tendons wrapping around bone.



Question (†† **Vertical cylinder is optional**)	Horizontal cylinder $g = 0$ m/s^2	Horizontal cylinder $g = 9.8$ m/s^2	Vertical cylinder $g = 9.8$ m/s^2
Time for Q to hit the cylinder (3 digits):	2.041666 sec	1.536 sec	1.7259 sec
Is string tension always positive?	**Yes**/No	**Yes**/No	**Yes**/No
Is ${}^N\vec{\mathbf{v}}^Q$ always perpendicular to string tension?	**Yes**/No	**Yes**/No	**Yes**/No
Is Q's $K + U$ conserved? (kinetic plus gravitational potential energy)	**Yes**/No	**Yes**/No	**Yes**/No
Is Q's speed in N constant?	**Yes**/No	Yes/**No**	Yes/**No**
Is Q's angular momentum about N_o conserved?	Yes/**No**	Yes/**No**	Yes/**No**
Is Q's angular momentum about B_N conserved? (B_N is the point of N touching the unwrapped string)	Yes/**No**	Yes/**No**	Yes/**No**



Horizontal cylinder: Trajectory for $g = 9.8\ \frac{\text{m}}{\text{s}^2}$



Vertical cylinder: Trajectory for $g = 9.8\ \frac{\text{m}}{\text{s}^2}$

Solution at www.MotionGenesis.com ⇒ Get Started ⇒ **Pendulum/Tether ball (wrap particle around cylinder).**