Concepts in Newtonian mechanics (average for Stanford graduate students = **35**%)

For each question, fill in the blank, circle true or false, circle one (or more) of the multiple choice answers, write the definition, or complete the drawing. The percentages of correct answers are from the pretest for 153 students in Stanford University's Advanced Dynamics classes in 2004, 2005, 2006, 2007, 2008, and Harvey Mudd students in 2006.

76% The ***good product rule for differentiation*** that works when u and v are scalars, vectors, or matrices is (circle the correct answer):

$$\boxed{\frac{d(u*v)}{dt} = \frac{du}{dt}*\mathrm{v} + \mathrm{u}*\frac{dv}{dt}} \qquad \frac{d(u*v)}{dt} = u*\frac{dv}{dt} + v*\frac{du}{dt} \qquad \frac{d(u*v)}{dt} = v*\frac{du}{dt} + u*\frac{dv}{dt}$$

97% Two properties (attributes) of a vector are magnitude and direction.

??%

A zero vector $\vec{\mathbf{0}}$ has a magnitude of	$\boxed{0}$/**1**/**2**/∞.
A zero vector $\vec{\mathbf{0}}$ has no direction.	**True**/$\boxed{\textbf{False}}$.
A zero vector $\vec{\mathbf{0}}$ is ***parallel*** to any vector $\vec{\mathbf{v}}$.	$\boxed{\textbf{True}}$/**False**.
A zero vector $\vec{\mathbf{0}}$ is ***perpendicular*** to any vector $\vec{\mathbf{v}}$.	$\boxed{\textbf{True}}$/**False**.

??% Circle the vector operations below (scalar multiplication, addition, dot-product, etc.) that are **defined** for a position vector $\vec{\mathbf{a}}$ (with **units** of m) and a velocity vector $\vec{\mathbf{b}}$ (with **units** of $\frac{\text{m}}{\text{sec}}$).

$\boxed{-\vec{\mathbf{a}}}$ $\qquad \boxed{5\,\vec{\mathbf{a}}}$ $\qquad \boxed{\vec{\mathbf{a}}\ /\ 5}$ $\qquad \vec{\mathbf{a}} + \vec{\mathbf{b}}$ $\qquad \boxed{\vec{\mathbf{a}} \cdot \vec{\mathbf{b}}}$ $\qquad \boxed{\vec{\mathbf{a}} \times \vec{\mathbf{b}}}$

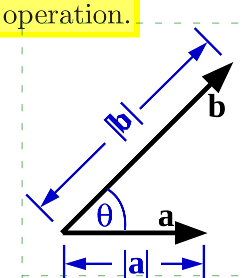
??% Consider the following process for solving the following vector equation for $\dot{\theta}$.
($\widehat{\mathbf{a}}_{\text{x}}$ is a unit vector and v_x, $\dot{\theta}$, and R are scalars).

$$v_x\,\widehat{\mathbf{a}}_{\text{x}} = \dot{\theta}\,R\,\widehat{\mathbf{a}}_{\text{x}} \qquad \Rightarrow \qquad \dot{\theta} = \frac{v_x\,\widehat{\mathbf{a}}_{\text{x}}}{R\,\widehat{\mathbf{a}}_{\text{x}}} = \frac{v_x}{R}$$

This process is a valid way to solve for $\dot{\theta}$. **True**/$\boxed{\textbf{False}}$.
Explain: It is incorrect to divide by a vector. Vector division is not a defined operation.

78% Write the ***definition*** of the dot-product of a vector $\vec{\mathbf{a}}$ with a vector $\vec{\mathbf{b}}$. Include a ***sketch*** with ***each symbol*** in the right-hand-side of your definition clearly labeled. The sketch should include $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, $|\vec{\mathbf{a}}|$, $|\vec{\mathbf{b}}|$, ...
Result:

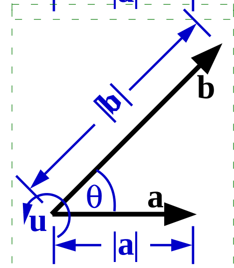
$$\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \triangleq |\vec{\mathbf{a}}|\,|\vec{\mathbf{b}}|\,\cos(\theta)$$



41% Write the ***definition*** of the cross-product of a vector $\vec{\mathbf{a}}$ with a vector $\vec{\mathbf{b}}$. Include a ***sketch*** with ***each symbol*** in your definition labeled and described.
Result:

$$\vec{\mathbf{a}} \times \vec{\mathbf{b}} \triangleq |\vec{\mathbf{a}}|\,|\vec{\mathbf{b}}|\ \sin(\theta)\,\widehat{\mathbf{u}}$$

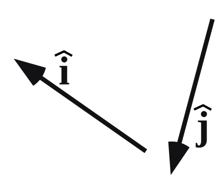
where $\widehat{\mathbf{u}}$ is a **unit vector** perpendicular to both $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ in a direction defined by the **right-hand rule** and θ is the angle between $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$.



72% For arbitrary non-zero vectors $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, $\vec{\mathbf{c}}$: $\quad \vec{\mathbf{a}} \cdot (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = (\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \cdot \vec{\mathbf{c}}$ **Never**/**Sometimes**/$\boxed{\textbf{Always}}$
A property of the ***scalar triple product*** is $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} \times \vec{\mathbf{a}} = 0$. $\boxed{\textbf{True}}$/**False**.

??% Knowing the angle between a unit vector $\widehat{\mathbf{i}}$ and unit vector $\widehat{\mathbf{j}}$ is 110°, calculate a numerical value for the magnitude of $\vec{\mathbf{v}} = 3\,\widehat{\mathbf{i}} + 4\,\widehat{\mathbf{j}}$.
Result:

$$|\vec{\mathbf{v}}| = 4.097745$$



??% The ***cross product*** of vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ can be written in terms of a real scalar s as $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = s\,\hat{\mathbf{u}}$ where $\hat{\mathbf{u}}$ is a unit vector perpendicular to both $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ in a direction defined by the **right-hand rule**. The coefficient s of the unit vector $\hat{\mathbf{u}}$ is inherently non-negative. **True**/**False**.

41% **Form the *unit* vector $\hat{\mathbf{u}}$ having the same direction as $c\,\hat{\mathbf{a}}_x$ (c is a non-zero real number).**

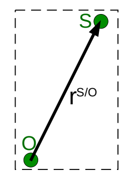
Result: $\hat{\mathbf{u}} = \dfrac{c}{\text{abs}(c)}\,\hat{\mathbf{a}}_x$ Note: $\hat{\mathbf{a}}_x$ is a unit vector and c is a non-zero real number, e.g., 3 or -3

42% The column matrix $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is identical to the vector $\hat{\mathbf{a}}_x + 2\,\hat{\mathbf{a}}_y + 3\,\hat{\mathbf{a}}_z$. **True**/**False**.

??% The following vector and column matrix addition produce equivalent results. **True**/**False**.
Note: $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ and $\hat{\mathbf{b}}_x$, $\hat{\mathbf{b}}_y$, $\hat{\mathbf{b}}_z$ are sets of orthogonal unit vectors.

$$\hat{\mathbf{a}}_x + 2\,\hat{\mathbf{a}}_y + 3\,\hat{\mathbf{a}}_z \quad + \quad 4\,\hat{\mathbf{b}}_x + 5\,\hat{\mathbf{b}}_y + 6\,\hat{\mathbf{b}}_z \quad = \quad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix}$$

Explain: In general, $\hat{\mathbf{a}}_i \neq \hat{\mathbf{b}}_x$ $(i = x, y, z)$, and vectors do not add like column matrices.



??% **Draw** $\vec{\mathbf{r}}^{\,S/O}$, the position vector of an object S from a point O.
In general and **without ambiguity**, S could be a (circle all appropriate objects):

Scalar	Real number	Complex number	Center of a circle
Vector	Point	Reference Frame	Mass center of a set of particles
Matrix	Set of Points	Rigid Body	Mass center of a rigid body
Line	Particle	Flexible Body	Set of flexible bodies
Orthogonal unit basis	Set of Particles	Set of Rigid bodies	System of particles and bodies

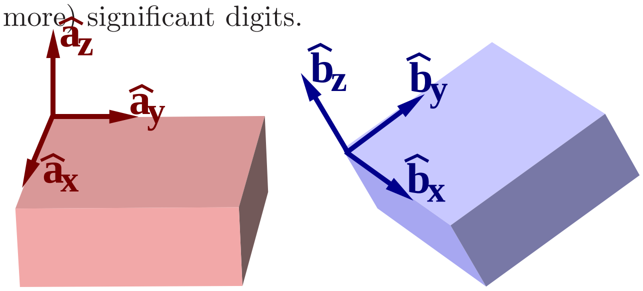
72% The following rotation matrix R relates two right-handed, orthogonal, unitary bases.
Calculate its inverse by-hand (no calculator) in less than 30 seconds.

$$R = \begin{bmatrix} 0.3830 & -0.6634 & 0.6428 \\ 0.9237 & 0.2795 & -0.2620 \\ -0.0058 & 0.6941 & 0.7198 \end{bmatrix} \quad \Rightarrow \quad R^{-1} = \begin{bmatrix} 0.3830 & 0.9237 & -0.0058 \\ -0.6634 & 0.2795 & 0.6941 \\ 0.6428 & -0.2620 & 0.7198 \end{bmatrix}$$

??% The following ${}^{a}R^{b}$ rotation table relates right-handed, orthogonal, unit vectors $\hat{\mathbf{a}}_x, \hat{\mathbf{a}}_y, \hat{\mathbf{a}}_z$ and $\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$.
Calculate the angle between $\hat{\mathbf{a}}_x$ and $\hat{\mathbf{b}}_z$ to four (or more) significant digits.

${}^{a}R^{b}$	$\hat{\mathbf{b}}_x$	$\hat{\mathbf{b}}_y$	$\hat{\mathbf{b}}_z$
$\hat{\mathbf{a}}_x$	0.9622502	-0.08418598	0.258819
$\hat{\mathbf{a}}_y$	0.1700841	0.9284017	-0.3303661
$\hat{\mathbf{a}}_z$	-0.2124758	0.3619158	0.9076734

$\angle(\hat{\mathbf{a}}_x, \hat{\mathbf{b}}_z) = 75^\circ$



46% The following vectors are expressed in terms of the orthogonal unit vectors $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ and t time.
Circle the vectors that can be differentiated without consideration of a reference frame.

$\vec{0}$	$2\,\hat{\mathbf{a}}_x + 4\,\hat{\mathbf{a}}_y$	$2\,\hat{\mathbf{a}}_x + t\,\hat{\mathbf{a}}_y$
$\hat{\mathbf{a}}_x$	$2\,\hat{\mathbf{a}}_x + 4\,\hat{\mathbf{a}}_y + 6\,\hat{\mathbf{a}}_z$	$2\,\hat{\mathbf{a}}_x + t\,\hat{\mathbf{a}}_y + \sin(t)\,\hat{\mathbf{a}}_z$

69% The definition of angular velocity of $\vec{\boldsymbol{\omega}} \triangleq \dot{\theta}\,\vec{\mathbf{k}}$ is a functional operational definition, i.e., in general, it is useful for calculating angular velocity and proving its properties (2D or 3D). **True**/**False**

19% ${}^N\vec{\boldsymbol{\omega}}^S$, the angular velocity of an object S in a reference frame N is to be determined.
In general and **without ambiguity**, S could be a (circle ***all*** appropriate objects):

Real number	Point	$\boxed{\text{Reference Frame}}$	Mass center of a set of particles
Vector	Set of Points	$\boxed{\text{Rigid Body}}$	Mass center of a rigid body
Matrix	Particle	Flexible Body	Set of flexible bodies
$\boxed{\text{Orthogonal unit basis}}$	Set of Particles	Set of Rigid bodies	System of particles and bodies

Repeat for ${}^N\vec{\boldsymbol{\alpha}}^S$, the angular acceleration of an object S in a reference frame N $\boxed{\text{box appropriate objects}}$.

9% ${}^N\vec{\mathbf{v}}^S$, the velocity of an object S in a reference frame N is to be determined.
In general and **without ambiguity**, S should be a (circle ***all*** appropriate objects):

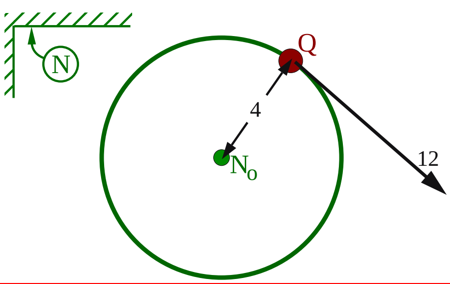
Vector	$\boxed{\text{Point}}$	Reference Frame	$\boxed{\text{Center of mass of a set of particles}}$
Matrix	Set of Points	Rigid Body	$\boxed{\text{Center of mass of a rigid body}}$
$\boxed{\text{Center of a circle}}$	$\boxed{\text{Particle}}$	Flexible Body	Set of flexible bodies
Orthogonal unit basis	Set of Particles	Set of Rigid bodies	System of particles and bodies

Repeat for ${}^N\vec{\mathbf{a}}^S$, the acceleration of an object S in a reference frame N $\boxed{\text{box appropriate objects}}$.

44% The following figures show a point Q moving in a **plane** N. Point N_o is fixed in N. The left-figure shows Q moving clockwise with speed 12 on a circle of radius 4 (the circle is fixed in N and centered at N_o). The right-figure shows Q moving with a speed of 12 on a horizontal line that is a distance 4 from N_o. **Box** the following true statements about a uniquely-defined ***angular velocity*** for Q.

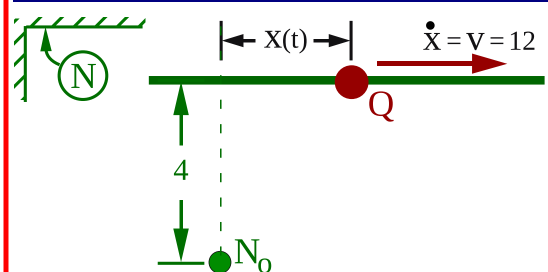
Left figure:

Q's angular velocity in N is $\vec{\mathbf{0}}$.
Q's angular velocity in N is a non-zero vector.
Q's angular velocity in N is $\vec{\infty}$.
$\boxed{\text{$Q$'s angular velocity in N does not exist.}}$



Right figure:

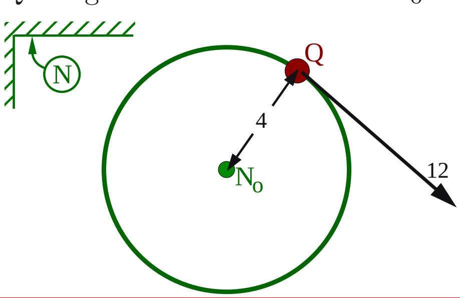
Q's angular velocity in N is $\vec{\mathbf{0}}$.
Q's angular velocity in N is a non-zero vector.
Q's angular velocity in N is $\vec{\infty}$.
$\boxed{\text{$Q$'s angular velocity in N does not exist.}}$



44% The following figures show a particle Q of mass 1 kg moving in a **plane** N. Point N_o is fixed in N. The figure on the left shows Q moving clockwise with speed 12 on a circle of radius 4 that is centered at N_o. The figure on the right shows Q moving with a speed of 12 on a horizontal line that is 4 from N_o. **Box** the following true statements about Q's ***angular momentum*** in N.

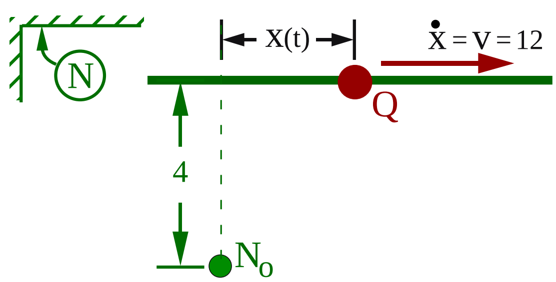
Left figure:

Q's angular momentum about N_o is $\vec{\mathbf{0}}$.
$\boxed{\text{$Q$'s angular momentum about N_o is not $\vec{\mathbf{0}}$.}}$
Q's angular momentum about N_o is $\vec{\infty}$.
Q's angular momentum about N_o does not exist.

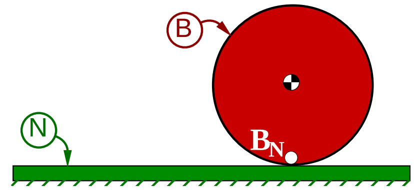


Right figure:

Q's angular momentum about N_o is $\vec{\mathbf{0}}$.
$\boxed{\text{$Q$'s angular momentum about N_o is not $\vec{\mathbf{0}}$.}}$
Q's angular momentum about N_o is $\vec{\infty}$.
Q's angular momentum about N_o does not exist.



51% The figure to the right shows a thin circular disk B that remains in contact with a horizontal plane N. The point of B in contact with N at the instant this picture was taken is denoted B_N. For the questions that follow, regard rolling and sliding to imply some kind of motion of B in N and assume any rotation of B in N is perpendicular to the circular portion of B



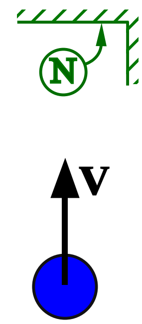
When B ***slides*** on N, the velocity of B_N in N ***must be*** zero. **True/$\boxed{\text{False}}$**

When B ***rolls*** on N, the velocity of B_N in N ***must be*** zero. **$\boxed{\text{True}}$/False**

When B ***slides*** on N, the acceleration of B_N in N ***can be*** zero. **$\boxed{\text{True}}$/False**

When B ***rolls*** on N, the acceleration of B_N in N ***can be*** zero. **True/$\boxed{\text{False}}$**

??% The vector $\vec{\mathbf{v}}$ measures the velocity of a baseball (particle) thrown straight upward on Earth (a Newtonian reference frame N). When the ball reaches maximum height, $\vec{\mathbf{v}} = \vec{\mathbf{0}}$. Knowing $\vec{\mathbf{v}} = \vec{\mathbf{0}}$ when the ball reaches maximum height and Earth's gravitational acceleration constant $g \approx 9.8 \frac{\text{m}}{\text{s}^2}$, decide if the following statement about $\vec{\mathbf{a}}$ (the ball's acceleration in N) is true. If false, box the incorrect part of the statement.



$$\vec{\mathbf{a}} \triangleq \boxed{\frac{{}^N d\vec{\mathbf{v}}}{dt} = \frac{{}^N d\,(\vec{\mathbf{0}})}{dt}} = \frac{d\,\vec{\mathbf{0}}}{dt} = \vec{\mathbf{0}} \qquad \textbf{True/}\boxed{\textbf{False}}$$

Explain: It is incorrect to time-differentiate the instantaneous value $\vec{\mathbf{v}} = \vec{\mathbf{0}}$. Time-differentiation must occur over dt, a non-zero ***interval*** of t (not at an ***instant***).

33% ${}^N K^S$, the ***kinetic energy*** of an object S in a reference frame N is to be determined. Objects S that can have a non-zero kinetic energy are (circle ***all*** appropriate objects):

Complex number	Point	Reference Frame	Center of mass of a set of particles
Vector	Set of Points	$\boxed{\text{Rigid Body}}$	Center of mass of a rigid body
Matrix	$\boxed{\text{Particle}}$	$\boxed{\text{Flexible Body}}$	$\boxed{\text{Set of flexible bodies}}$
Orthogonal unit basis	$\boxed{\text{Set of Particles}}$	$\boxed{\text{Set of Rigid bodies}}$	$\boxed{\text{System of particles and bodies}}$

Repeat for ${}^N\vec{\text{L}}^S$, the ***linear momentum*** of object S in reference frame N $\boxed{\text{box appropriate objects}}$.

71%, 53% Kinetic energy of a system S in a reference frame N always exists. **$\boxed{\text{True}}$/False**
Potential energy of a system S in a reference frame N always exists. **True/$\boxed{\text{False}}$**

??%

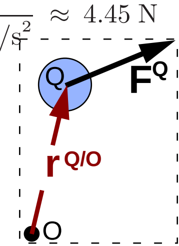
1 Newton is defined as (circle all that apply)	$\boxed{1 \frac{\text{kg m}}{\text{s}^2}}$ $9.81 \frac{\text{kg m}}{\text{s}^2}$ $32.2 \frac{\text{kg m}}{\text{s}^2}$	None of these	
1 lb_f is defined as or approximately equal to (circle all that apply)	$1 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2}$	$\boxed{1 \text{ slug} \cdot \frac{\text{ft}}{\text{s}^2}}$	$1 \text{ lb}_\text{m} \cdot \frac{\text{ft}}{\text{s}^2}$
	$9.81 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2}$	$9.81 \text{ slug} \cdot \frac{\text{ft}}{\text{s}^2}$	$9.81 \text{ lb}_\text{m} \cdot \frac{\text{ft}}{\text{s}^2}$
	$32.2 \text{ kg} \cdot \frac{\text{m}}{\text{s}^2}$	$32.2 \text{ slug} \cdot \frac{\text{ft}}{\text{s}^2}$	$\boxed{32.2 \text{ lb}_\text{m} \cdot \frac{\text{ft}}{\text{s}^2}}$

Using the exact Section 22.6 NIST conversion factor for lbm to kg and the exact conversion factor 1 inch $\triangleq$ 2.54 cm, show how to calculate the conversion factor for lbf to Newton.

Result

$$1 \text{ lbf} \approx \frac{32.2 \text{ lbm ft}}{\text{s}^2} * \frac{0.45359237 \text{ kg}}{1 \text{ lbm}} * \frac{12 \text{ inch}}{1 \text{ ft}} * \frac{2.54 \text{ cm}}{1 \text{ inch}} * \frac{1 \text{ m}}{100 \text{ cm}} * \frac{1 \text{ N}}{1 \text{ kg m/s}^2} \approx 4.45 \text{ N}$$

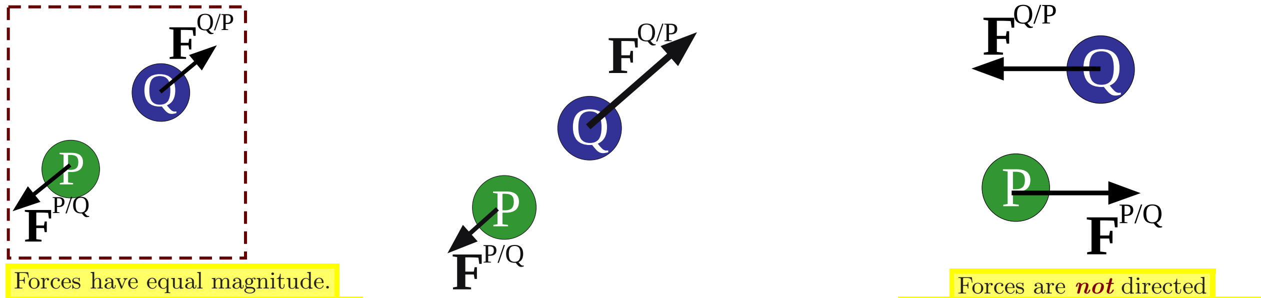
68% Write the ***definition*** for the moment of force $\vec{\mathbf{F}}^Q$ applied to point Q about point O. Draw a sketch with *each* part of your definition clearly labeled.



Result:

$$\vec{\mathbf{M}}^{\vec{\mathbf{F}}^Q/O} \triangleq \vec{\mathbf{r}}^{Q/O} \times \vec{\mathbf{F}}^Q$$

??% Circle the forces that obey the ***law of action/reaction***. **Explain** why each pair obeys/disobeys.



Forces have equal magnitude.
Forces have opposite directions.
Forces are directed towards each other.

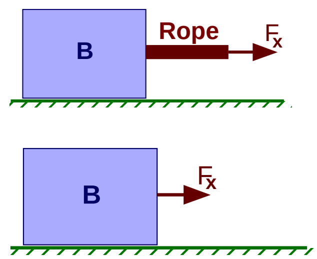
Force magnitudes are ***unequal***.

Forces are ***not*** directed towards or away from each other (different lines of action).

??% The following figure shows an inextensible rope attached to the right-side of a metallic particle B that is in contact with a **rough** flat horizontal magnetic table (a Newtonian reference frame). A horizontal force with measure F_x is applied to the distal end of the rope.

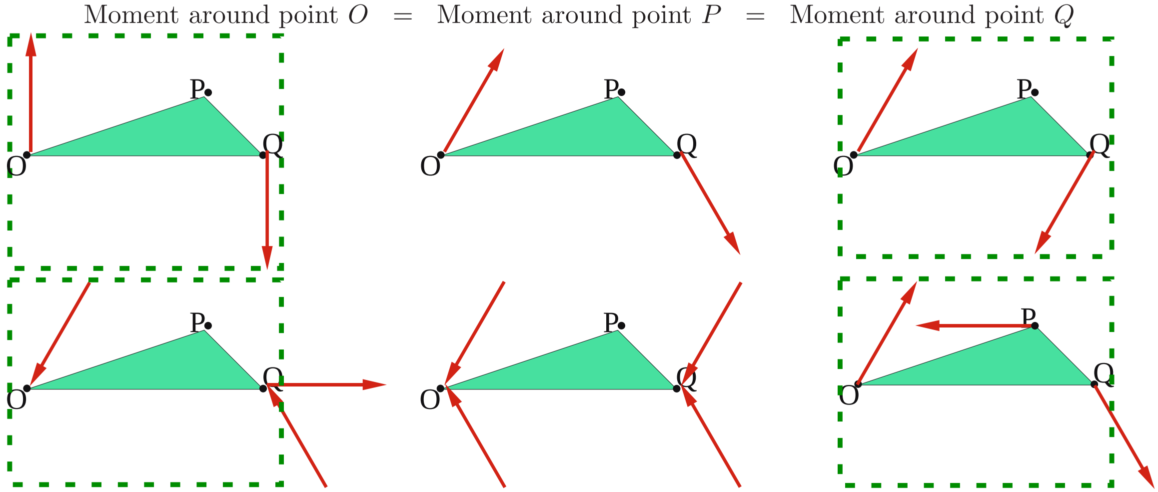
For each analysis below and **ignoring gravity**, decide whether it makes a difference to the block's motion or forces if F_x acts through the rope (top-right figure) or directly on B (bottom-right figure).

Mass of rope	Static/dynamic analysis	Makes a difference?
Massless	B and rope are stationary (statics)	**Yes/$\boxed{\textbf{No}}$**
Massive	B and rope are stationary (statics)	**Yes/$\boxed{\textbf{No}}$**
Massless	B and rope translate right at same constant speed	**Yes/$\boxed{\textbf{No}}$**
Massive	B and rope translate right at same constant speed	**Yes/$\boxed{\textbf{No}}$**
Massless	B and rope translate right at variable speeds	**Yes/$\boxed{\textbf{No}}$**
Massive	B and rope translate right at variable speeds	**$\boxed{\textbf{Yes}}$/No**



11% Consider the six figures below, each which contain a set of forces. Circle the figure(s) in which the moment of its set of forces about points O, P, and Q all are equal, i.e.,

Moment around point O = Moment around point P = Moment around point Q



Note: All forces have the same magnitude. Forces that are not horizontal or vertical are 30° from vertical.

75% All torques are moments. **$\boxed{\textbf{True}}$/False**

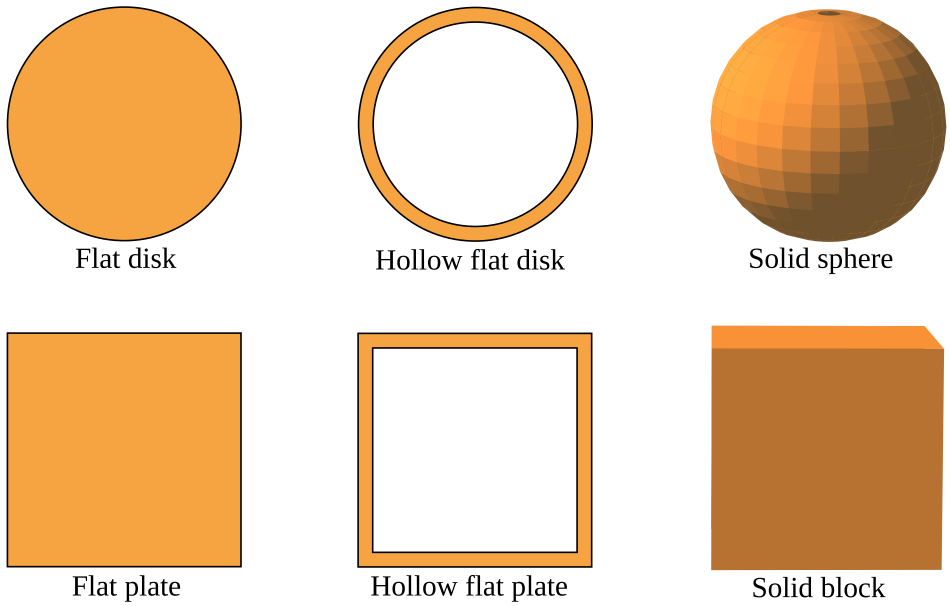
61% All moments are torques. **True/$\boxed{\textbf{False}}$**

61% The moment of a couple about a point O is equal to the moment of the couple about any other point P **$\boxed{\textbf{True}}$/False**

51%
$\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ is useful for analyzing 3D translational motions of a rigid body. **$\boxed{\textbf{True}}$/False**
$\vec{\mathbf{M}} = \mathrm{I}\,\vec{\boldsymbol{\alpha}}$ is useful for analyzing 3D rotational motions of a rigid body. **True/$\boxed{\textbf{False}}$**

57% **Conceptual example of moments of inertia**

Each object below has a uniform density and an equal mass. After identifying the mass center of each figure with an ◐, answer the following questions about I_{zz}, the moment of inertia of each object about the line that passes through its mass center and is perpendicular to the plane of the paper.



(a) Consider the first row of objects. The **flat disk/hollow disk/solid sphere** has the **largest** value of I_{zz}, whereas the **flat disk/hollow disk/solid sphere** has the **smallest** value of I_{zz}.

(b) Consider the second row of objects. The **flat plate/hollow plate/solid block** has the **largest** value of I_{zz}, whereas the flat plate and solid block have **equal** values of I_{zz}.

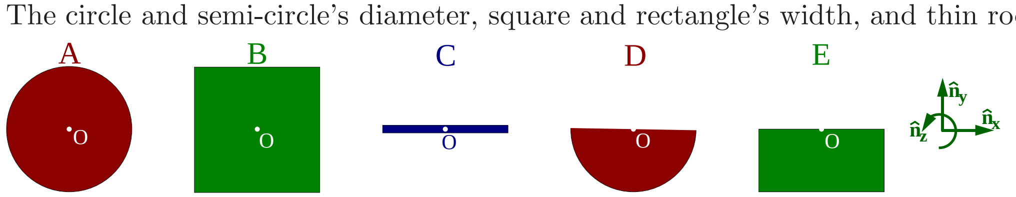
(c) Consider all the objects in both rows. The hollow plate has the **largest** value of I_{zz}, whereas the solid sphere has the **smallest** value of I_{zz}.

??% **Concepts: What objects have a moment of inertia?**

Consider the ***moment of inertia*** $\mathrm{I}^{S/O}_{\hat{\mathbf{u}}\,\hat{\mathbf{u}}}$ of an object S about a point O for the unit vector $\hat{\mathbf{u}}$.
In general, for $\mathrm{I}^{S/O}_{\hat{\mathbf{u}}\,\hat{\mathbf{u}}}$ to be a positive real number, S should be a (circle ***all*** appropriate objects):

Complex number	Point	Reference Frame	Center of mass of a set of particles
Vector	Set of Points	Rigid Body	Center of mass of a rigid body
Matrix	Particle	Flexible Body	Set of flexible bodies
Orthogonal unit basis	Set of Particles	Set of Rigid bodies	System of particles and bodies

0% Objects A, B, C, D, and E are all flat planar objects with uniform density and the ***same*** mass. The circle and semi-circle's diameter, square and rectangle's width, and thin rod's length are ***equal***.



S denotes one of A, B, C, D, or E.

36% Consider $\mathrm{I}^{S/O}_{\mathrm{zz}}$, S's moment of inertia about the line passing through point O and parallel to $\hat{\mathbf{n}}_{\mathrm{z}}$. Knowing moment of inertia is mass $*$ distance^2, use **visual estimates** to list the objects in ascending order of $\mathrm{I}^{S/O}_{\mathrm{zz}}$. If two objects have the same value of $\mathrm{I}^{S/O}_{\mathrm{zz}}$, group them together.

Result:	Smallest	C		D, A		E, B	Largest

28% Consider $I_{zz}^{S/S_{cm}}$, S's moment of inertia about the line passing through S_{cm} (the mass center of S) and parallel to $\hat{\mathbf{n}}_z$. Use visual estimates to list the objects in ascending order of $I_{zz}^{S/S_{cm}}$.
Note: A and E have nearly equal $I_{zz}^{S/S_{cm}}$. The textbook's inertia appendix helps resolve their difference.

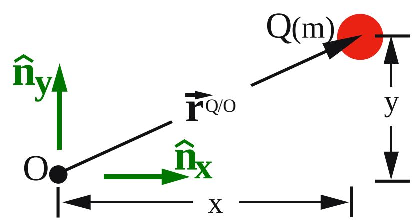
Result:	Smallest	D (given)	C	E	A	B	Largest

23% Consider $I_{xy}^{S/O}$, S's product of inertia for point O and unit vectors $\hat{\mathbf{n}}_x$ and $\hat{\mathbf{n}}_y$. For each object, visually determine if $I_{xy}^{S/O}$ is negative (−), zero (0), or positive (+).

Result:	A	B	C	D	E
	− $\boxed{0}$ +	− $\boxed{0}$ +	− $\boxed{0}$ +	− $\boxed{0}$ +	− $\boxed{0}$ +

16% Moments of inertia of a particle

The figure shows a particle Q of mass m and right-handed orthogonal unit vectors $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$. Q's position vector from a point O is $x\,\hat{\mathbf{n}}_x + y\,\hat{\mathbf{n}}_y + z\,\hat{\mathbf{n}}_z$.



Express I_{xx} (Q's ***moment of inertia*** about O for $\hat{\mathbf{n}}_x$) in terms of some or all of m, x, y z. Similarly for I_{yy} and I_{zz}.

Express I_{xy} (Q's ***product of inertia*** about O for $\hat{\mathbf{n}}_x$ and $\hat{\mathbf{n}}_y$) in terms of some or all of m, x, y z. Similarly for I_{xz} and I_{yz}.

Result:

$\boxed{I_{xx} = m\,(y^2 + z^2)}$ $\boxed{I_{yy} = m\,(x^2 + z^2)}$ $\boxed{I_{zz} = m\,(x^2 + y^2)}$

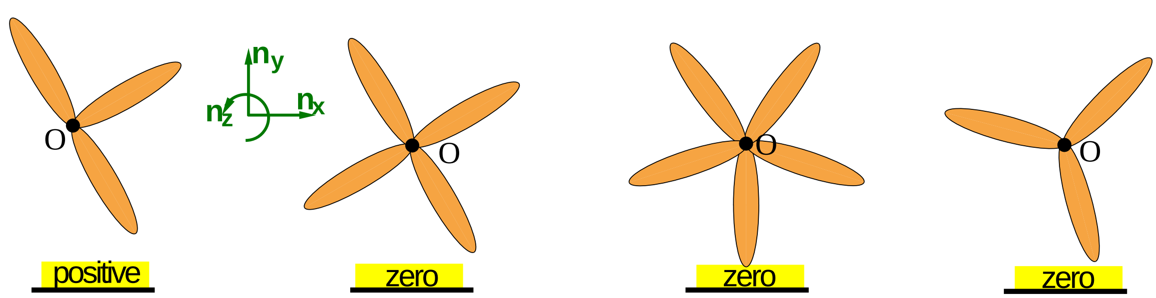
$\boxed{I_{xy} = -m\;x\;y}$ $\boxed{I_{xz} = -m\,x\,z}$ $\boxed{I_{yz} = -m\,y\,z}$

Circa 1895, Gibbs invented the ***inertia dyadic*** as a **convenient "suitcase"** for holding moments and products of inertia. Write Q's inertia dyadic about O in terms of $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ and I_{ij} $(i, j = x, y, z)$. If needed, refer to Section 17.1.

$$\boxed{\begin{aligned} \vec{\vec{\mathbf{I}}} = {} & I_{xx}\,\hat{\mathbf{n}}_x\,\hat{\mathbf{n}}_x + I_{xy}\,\hat{\mathbf{n}}_x\,\hat{\mathbf{n}}_y + I_{xz}\,\hat{\mathbf{n}}_x\,\hat{\mathbf{n}}_z \\ & + I_{xy}\,\hat{\mathbf{n}}_y\,\hat{\mathbf{n}}_x + I_{yy}\,\hat{\mathbf{n}}_y\,\hat{\mathbf{n}}_y + I_{yz}\,\hat{\mathbf{n}}_y\,\hat{\mathbf{n}}_z \\ & + I_{xz}\,\hat{\mathbf{n}}_z\,\hat{\mathbf{n}}_x + I_{yz}\,\hat{\mathbf{n}}_z\,\hat{\mathbf{n}}_y + I_{zz}\,\hat{\mathbf{n}}_z\,\hat{\mathbf{n}}_z \end{aligned}}$$

8% Conceptual example of products of inertia

The following shows four uniform-density objects. For each object, consider I_{xy}, the product of inertia of the object for lines that pass through point O and are parallel to $\hat{\mathbf{n}}_x$ and $\hat{\mathbf{n}}_y$. Below each object, mark whether the product of inertia is ***negative***, ***zero***, or ***positive***.[3]



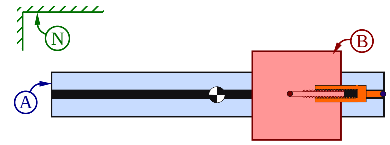
32% Conceptual example of translational motion

A rigid body B is connected to a rigid body A with a force/linear actuator. Initially, A and B are **at rest** (stationary) in deep empty space in a Newtonian (inertial) reference frame N.

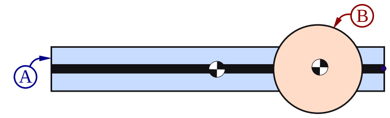
[3]**Instructor Note:** Product of inertia conventions allow for valid answers for the previous question as $-m\,x\,y$ or $+m\,x\,y$. This question was marked correct as long as the answers were consistent.

Is it possible for the force actuator to move:

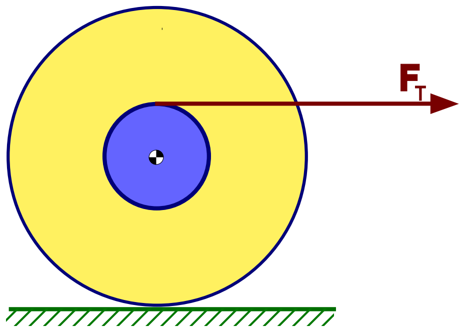
- A's mass center in N? $\boxed{\textbf{Yes}}$/**No**
- B's mass center in N? $\boxed{\textbf{Yes}}$/**No**
- the system's mass center in N? **Yes**/$\boxed{\textbf{No}}$



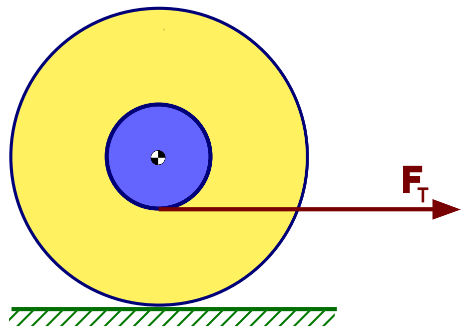
The previous 3 answers are the $\boxed{\textbf{same}}$/**different** if B is connected to A with a torque/rotational motor (instead of a force actuator).



??% Each rigid wheel below has a rope that is wrapped around its axle which is pulled with tension F_T. The wheel starts from rest and is constrained to **roll** with a simple angular velocity (Section 12.12 discusses rolling). Determine which way the wheel rolls.



The wheel **rolls** **left**/$\boxed{\textbf{right}}$



The wheel **rolls** **left**/$\boxed{\textbf{right}}$

Note: This problem was motivated by the dynamics of the old-fashioned penny farthing bicycle.
This "Which Way Will It Roll" puzzle was the Sept. 19, 2011 Wordplay (New York Times crossword blog).
Video: Search YouTube with "Veritasium spool" or visit http://www.youtube.com/watch?v=Bwf3msm7rqM