Important Concepts in Dynamics

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Adapted from

Mitiguy, P. *Advanced Dynamics & Motion Simulation* and Levinson, D. *Dynamics for Engineering Analysis*

Point := a *location* in space with no spatial dimension (i.e., no height, width, or depth; and consequently no mass). A point's location can be measured with a position vector that characterizes its position from another point.

Particle := a point with mass

Vector := a mathematical entity defined as a quantity having magnitude and direction. A vector's *magnitude* is a real non-negative number. A vector's *direction* can be resolved into ***orientation*** (e.g., like east-west alignment) and ***sense*** (which way the vector is pointing along its orientation)

Position vector := a vector that relates the location of two points. Has the units of length

Vector basis := a set of linearly independent vectors that 'span' a space. Linearly independent means that no one of the basis vectors can be constructed as a linear combinations of the others. Span means every vector of the space(i.e., 1D, 2D, 3D space) can be expressed uniquely as a linear combination of the basis vectors. (See also: https://en.wikipedia.org/wiki/Basis_(linear_algebra)). To span 3D space, three non-coplanar vectors are needed. Note that a vector basis is *not* associated with a particular point of a reference frame, therefore the tails of basis vectors do not need to all touch as if to suggest an 'origin'. A basis vector is a 'free' vector that neither has a point of application nor a line of action associated with it.

Rigid vector basis := a vector basis where the magnitude of the basis vectors is constant and the angles between basis vectors are constant (not necessarily orthogonal). With two 3D rigid vector bases, one may measure relative orientation, angular velocity, and angular acceleration. A rigid vector basis is *not* associated with a particular point of a reference frame.

Non-rigid vector basis := a vector basis where the magnitude of one of the basis vectors is variable or the angle between two basis vectors is variable.

Right-handed orthogonal unitary vector basis := a vector basis where the basis vectors are unit vectors that are mutually perpendicular and have the intrinsic order, $\hat{a}_x \times \hat{a}_y = \hat{a}_z \;\; \hat{a}_y \times \hat{a}_z = \hat{a}_x \;\; \hat{a}_z \times \hat{a}_x = \hat{a}_y$

Non-orthogonal basis := a vector basis where the basis vectors are not orthogonal.

Reference frame := a rigid 3D object in which points, curves, surfaces, unit vectors, rigid vector bases, rigid frames, rigid bodies, and other rigid objects can be fixed. A reference frame can be constructed with as few as three non-collinear points whose distances from each other are *constant*. Every geometric solid can be regarded as reference frame. All rigid bodies are reference frames, but not all reference frames are rigid bodies. A reference frame extends infinitely in all spatial directions. It is not necessary that something be a material object in order for it to be a reference frame. For any one reference frame, there always exists infinitely many vector bases that can be associated with it. Using a

point and vector basis *fixed in a reference frame*, an arbitrary (and possibly moving point) Q's position, velocity, and acceleration can be measured.

The orientation of a rigid body B relative to a reference frame A is unique, but its measurement is not unique, because the analyst is free to fix rigid vector bases on B in an infinite number of ways. The measurement of B's orientation in A requires a rigid vector basis fixed in B and a rigid vector basis fixed in A. Choices of different rigid bases will lead to different measurements of B's unique orientation relative to A.

Rigid frame := a rigid vector basis with an associated point ("origin"). A rigid frame implies a unique reference frame. An infinite number of rigid frames can be fixed in a reference frame.

Newtonian reference frame := a non-rotating, non-accelerating reference frame in which $\vec{F} = m\vec{a}$ accurately predicts forces and motion.

Rigid body := a bound, interconnected aggregate of matter in which the distances between all points of the body are *constant*. A rigid body is a reference frame. A rigid body has mass and inertia.

Coordinate axes := three lines (axes) emanating from a point (origin) along which a set of coordinates are placed. Coordinates are scalar quantities that help locate points or orient objects (e.g., distances or angles). Coordinates systems may or may not be orthogonal, for example a Cartesian coordinate system assigns three numbers corresponding to locations along three mutually perpendicular axes (xyz). An infinite number of coordinate axis sets can be embedded in a rigid body or reference frame. The term reference frame and coordinate axes are *NOT* synonymous. It is also not appropriate to refer to basis vectors as 'coordinate axes' or vice versa.

System := a collection of objects, such as points, particles, rigid frames, rigid bodies

Rotation matrix := a matrix that relates orientation of two orthogonal unitary vector bases

Angular velocity := a vector that relates time-rate of change of *orientation* of two rigid bases (or rigid frames or reference frames or rigid bodies)

Angular acceleration := a vector that relates the time-rate of change of *angular velocity* of two rigid bases (or rigid frames or reference frames or rigid bodies)

Velocity := a vector that describes the time-rate of change of a point's *location* in a reference frame

Acceleration := a vector that describes the time-rate of change of a point's *velocity* in a reference frame

Newton's Axioms

1. Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it. (Restatement of Galileo's principle of inertia 1638)

2. The change of motion is proportional to the motive force impressed, and is made in the direction of the right line in which that force is impressed. $\vec{F} = m\vec{a}$ (How the momentum of a body changes (or its velocity, if the mass is constant) by the influence of forces, namely the resultant force on the body)

3. To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal, and *directed to contrary parts*.

