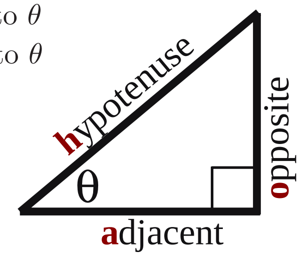
4.1 ♣ SohCahToa: Sine, cosine, tangent as ratios of sides of a right triangle. (Section 1.5)

The following shows a ***right triangle*** with one of its angles labeled as θ.
Write definitions for sine, cosine, and tangent in terms of:

- **h**ypotenuse – the triangle's longest side (opposite the 90° angle).
- **o**pposite – the side opposite to θ
- **a**djacent – the side adjacent to θ

Note: A mnemonic for these definitions is "*SohCahToa*".

Note: A ***right triangle*** is a triangle with a 90° angle.



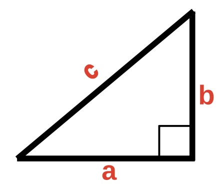
$$\sin(\theta) \triangleq \frac{\textit{opposite}}{\textit{hypotenuse}}$$

$$\cos(\theta) \triangleq \frac{\textit{adjacent}}{\textit{hypotenuse}}$$

$$\tan(\theta) \triangleq \frac{\textit{opposite}}{\textit{adjacent}} = \frac{\sin(\theta)}{\cos(\theta)}$$

4.2 ♣ Pythagorean theorem and law of cosines - memorize. (Section 1.5.1).

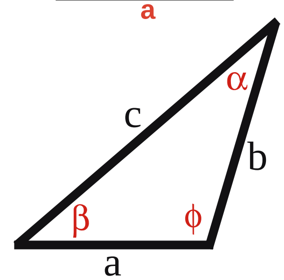
Draw a right-triangle with a hypotenuse of length c and other sides of length a and b. Relate c to a and b with the ***Pythagorean theorem***.



Result:

$$c^2 = a^2 + b^2$$

A non-right-triangle has angles α, β, ϕ opposite sides a, b, c, respectively.
Use the ***law of cosines*** to complete each formula below.



Result:

$$c^2 = a^2 + b^2 - 2\,a\,b\,\cos(\phi)$$

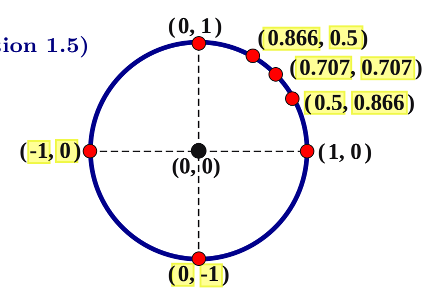
$$a^2 = b^2 + c^2 - 2\,b\,c\,\cos(\alpha)$$

$$b^2 = c^2 + a^2 - 2\,c\,a\,\cos(\beta)$$

The ***Pythagorean theorem*** is a special case of the ***law of cosines***. **True**/**False**. (circle one).

4.3 ♣ Memorize common sines and cosine. (Section 1.5)

$\sin(0°) = 0$	$\cos(0°) = 1$
$\sin(30°) = \frac{1}{2}$	$\cos(30°) = \frac{\sqrt{3}}{2}$
$\sin(45°) = \frac{\sqrt{2}}{2}$	$\cos(45°) = \frac{\sqrt{2}}{2}$
$\sin(60°) = \frac{\sqrt{3}}{2}$	$\cos(60°) = \frac{1}{2}$
$\sin(90°) = 1$	$\cos(90°) = 0$



Label the coordinates of each point on the unit circle.

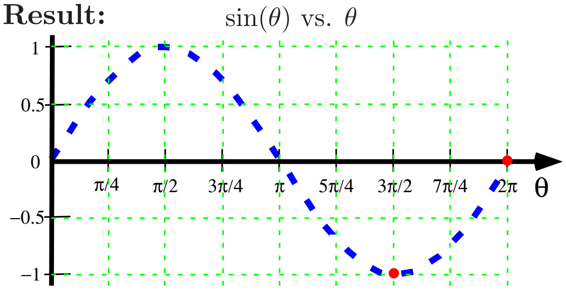
4.4 ♣ Graphing sine and cosine - (a now-obvious invention from 1730 A.D.) (Section 1.5.2)

Graph sine and cosine as functions of the angle θ in radians over the range $0 \le \theta \le 2\,\pi$.

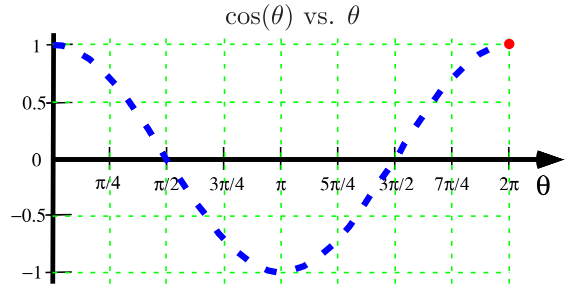
The mathematician Euler was first to regard sine and cosine as **functions** (not just ratios of sides of a triangle).

Result:

$\sin(\theta)$ vs. θ



$\cos(\theta)$ vs. θ



4.5 ♣ Ranges for arguments and return values for inverse trigonometric functions.

Determine all real return values and argument values for the following **real** trigonometric and inverse-trigonometric functions in computer languages such as Java, C^{++}, **M**otion**G**enesis, and MATLAB®.

Possible return values	Function	Possible argument values	Note
$-1 \le z \le 1$	$z = \cos(x)$	$-\infty < x < \infty$	
$-1 \le z \le 1$	$z = \sin(x)$	$-\infty < x < \infty$	
$-\infty < z < \infty$	$z = \tan(x)$	$-\infty < x < \infty$	$x \ne \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$
$0 \le z \le \pi$	$z = \text{acos}(x)$	$-1 \le x \le 1$	
$-\pi/2 \le z \le \pi/2$	$z = \text{asin}(x)$	$-1 \le x \le 1$	
$-\pi/2 < z < \pi/2$	$z = \text{atan}(x)$	$-\infty < x < \infty$	
$-\pi \le z \le \pi$	$z = \texttt{atan2}(y, x)$	$-\infty < y < \infty$ $-\infty < x < \infty$	$\texttt{atan2}(0,0)$ is undefined

4.6 ♣ What is an angle? (Section 5.6).

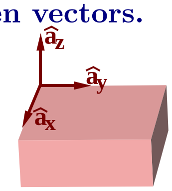
Draw the "geometry equipment" listed in the 1^{st} column of the following table.
Complete the 2^{nd} column with appropriate ranges for the angle θ (in degrees).

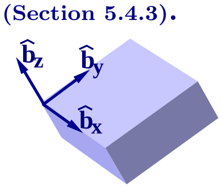
"Geometry equipment"	Draw	Appropriate range for θ
2 lines		$0° \le \theta \le 90°$
Vector and line		$0° \le \theta \le 90°$
2 vectors		$0° \le \theta \le 180°$
2 vectors and a sense of positive rotation		$-180° < \theta \le 180°$
2 vectors, a sense of $\pm$ rotation, and time-history/continuity	Not applicable	$-\infty° < \theta < \infty°$

4.7 Calculating dot-products, cross-products, and angles between vectors. (Section 5.4.3).

The ${}^aR^b$ rotation table relates two sets of right-handed, orthogonal, unit vectors, namely $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$ and $\hat{b}_x$, $\hat{b}_y$, $\hat{b}_z$.

${}^aR^b$	$\hat{b}_x$	$\hat{b}_y$	$\hat{b}_z$
$\hat{a}_x$	0.9623	−0.0842	0.2588
$\hat{a}_y$	0.1701	0.9284	−0.3304
$\hat{a}_z$	−0.2125	0.3619	0.9077





(a) Efficiently determine the following dot-products and angles between vectors (2^+ significant digits).
Then perform the following calculations involving $\vec{v}_1 = 2\,\hat{a}_x$ and $\vec{v}_2 = \hat{a}_x + \hat{b}_x$.

$\hat{a}_x \cdot \hat{a}_x = 1$	$\hat{a}_y \cdot \hat{a}_z = 0$	$\hat{b}_z \cdot \hat{b}_y = 0$
$\hat{a}_x \cdot \hat{b}_x = 0.9623$	$\hat{a}_x \cdot \hat{b}_y = -0.0842$	$\hat{b}_z \cdot \hat{a}_y = -0.3304$
$\angle(\hat{a}_y, \hat{a}_y) = 0°$	$\angle(\hat{b}_z, \hat{b}_x) = 90°$	
$\angle(\hat{a}_y, \hat{b}_y) = 21.81°$	$\angle(\hat{b}_y, \hat{a}_z) = 68.78°$	

Result: $\vec{v}_1 \cdot \vec{v}_2 = 3.925$ $\quad \angle(\vec{v}_1, \vec{v}_2) = 7.897°$

$$\vec{v}_1 \times \vec{v}_2 = 0.5176\,\hat{b}_y + 0.1684\,\hat{b}_z = 0.425\,\hat{a}_y + 0.3402\,\hat{a}_z$$

(b) Express the unit vector $\hat{u}$ in the direction of $3\,\hat{a}_z + 4\,\hat{b}_z$ in terms of $\hat{a}_z$ and $\hat{b}_z$.
Express $\vec{v} = \hat{a}_y + \hat{b}_y$ in terms of $\hat{a}_x$, $\hat{a}_y$, $\hat{a}_z$.

Result:

$$\hat{u} = 0.4386\,\hat{a}_z + 0.5848\,\hat{b}_z$$

$$\vec{v} = -0.0842\,\hat{a}_x + 1.928\,\hat{a}_y + 0.3619\,\hat{a}_z$$

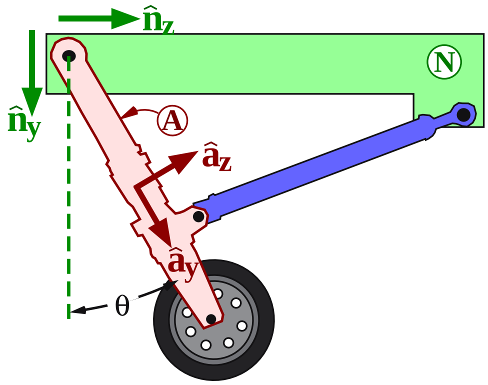
4.8 ♣ Efficient calculation of the inverse of a rotation matrix. (Section 5.4.2).

The following rotation matrix R relates two right-handed, orthogonal, unitary bases. Calculate its inverse by-hand (no calculator) in less than 30 seconds.

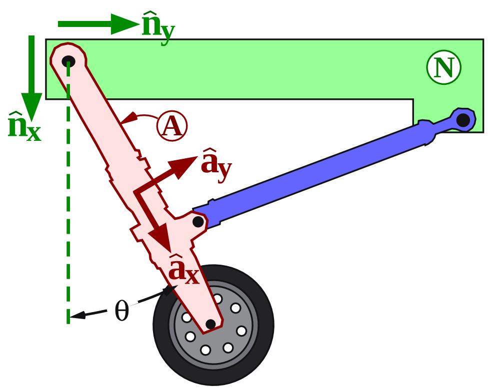
$$R = \begin{bmatrix} 0.3830 & -0.6634 & 0.6428 \\ 0.9237 & 0.2795 & -0.2620 \\ -0.0058 & 0.6941 & 0.7198 \end{bmatrix} \quad \Rightarrow \quad R^{-1} = \begin{bmatrix} 0.3830 & 0.9237 & -0.0058 \\ -0.6634 & 0.2795 & 0.6941 \\ 0.6428 & -0.2620 & 0.7198 \end{bmatrix}$$

4.9 SohCahToa: Rotation tables for a landing gear system. (Section 5.5).

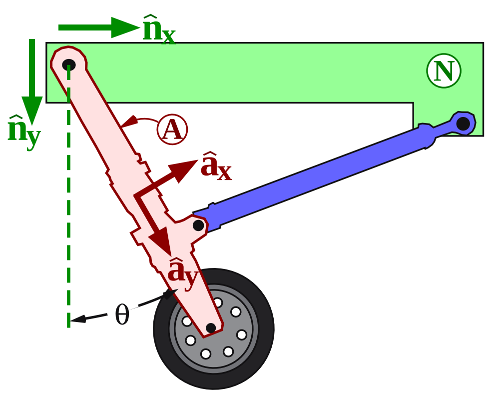
The figures below show three versions of the same landing gear system with a strut A that has a simple rotation relative to a fuselage N. Each figure has a different orientation for right-handed orthogonal unit vectors $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ (fixed in N) and $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ (fixed in A). **Redraw** $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$ and $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$ so it is **easy to see** sines and cosines. Then, form the ${}^{a}R^{n}$ rotation table for each figure.[1]



${}^{a}R^{n}$	$\hat{\mathbf{n}}_x$	$\hat{\mathbf{n}}_y$	$\hat{\mathbf{n}}_z$
$\hat{\mathbf{a}}_x$	1	0	0
$\hat{\mathbf{a}}_y$	0	$\cos(\theta)$	$\sin(\theta)$
$\hat{\mathbf{a}}_z$	0	$-\sin(\theta)$	$\cos(\theta)$



${}^{a}R^{n}$	$\hat{\mathbf{n}}_x$	$\hat{\mathbf{n}}_y$	$\hat{\mathbf{n}}_z$
$\hat{\mathbf{a}}_x$	$\cos(\theta)$	$\sin(\theta)$	0
$\hat{\mathbf{a}}_y$	$-\sin(\theta)$	$\cos(\theta)$	0
$\hat{\mathbf{a}}_z$	0	0	1



${}^{a}R^{n}$	$\hat{\mathbf{n}}_x$	$\hat{\mathbf{n}}_y$	$\hat{\mathbf{n}}_z$
$\hat{\mathbf{a}}_x$	$\cos(\theta)$	$-\sin(\theta)$	0
$\hat{\mathbf{a}}_y$	$\sin(\theta)$	$\cos(\theta)$	0
$\hat{\mathbf{a}}_z$	0	0	1

[1] Each figure has two missing vectors (e.g., $\hat{\mathbf{n}}_x$ and $\hat{\mathbf{a}}_x$ are missing from the first figure). Use the fact that each set of vectors is **right-handed** to add the missing vectors to each figure.