

Show work – except for ♣ fill-in-blanks.

4.1 ♣ **SohCahToa: Sine, cosine, tangent as ratios of sides of a right triangle.** (Section 1.4)

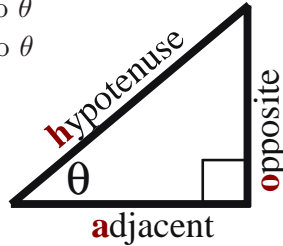
The following shows a *right triangle* with one of its angles labeled as θ .

Write definitions for sine, cosine, and tangent in terms of:

- **h**ypotenuse – the triangle’s longest side (opposite the 90° angle).
- **o**pposite – the side opposite to θ
- **a**djacent – the side adjacent to θ

Note: A mnemonic for these definitions is “SohCahToa”.

Note: A *right triangle* is a triangle with a 90° angle.



$$\sin(\theta) \triangleq \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos(\theta) \triangleq \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan(\theta) \triangleq \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin(\theta)}{\cos(\theta)}$$

4.2 ♣ **Pythagorean theorem and law of cosines - memorize.** (Section 1.4.1).

Draw a right-triangle with a hypotenuse of length c and other sides of length a and b . Relate c to a and b with the *Pythagorean theorem*.

Result:

$$c^2 = a^2 + b^2$$

A non-right-triangle has angles α, β, ϕ opposite sides a, b, c , respectively.

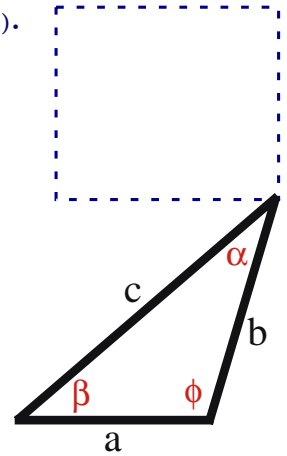
Use the *law of cosines* to complete each formula below.

Result:

$$c^2 = a^2 + b^2 - 2ab \cos(\phi)$$

$$a^2 = b^2 + c^2 - 2bc \cos(\alpha)$$

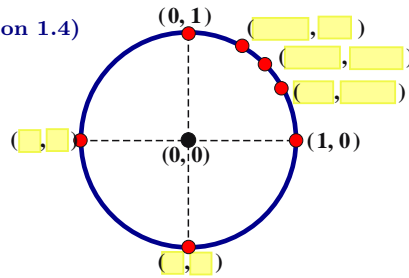
$$b^2 = a^2 + c^2 - 2ac \cos(\beta)$$



The *Pythagorean theorem* is a special case of the *law of cosines*. True/False. (circle one).

4.3 ♣ **Memorize common sines and cosine.** (Section 1.4)

$\sin(0^\circ) =$	<input type="text"/>	$\cos(0^\circ) =$	<input type="text"/>
$\sin(30^\circ) =$	<input type="text"/>	$\cos(30^\circ) =$	<input type="text"/>
$\sin(45^\circ) =$	<input type="text"/>	$\cos(45^\circ) =$	<input type="text"/>
$\sin(60^\circ) =$	<input type="text"/>	$\cos(60^\circ) =$	<input type="text"/>
$\sin(90^\circ) =$	<input type="text"/>	$\cos(90^\circ) =$	<input type="text"/>



Label the coordinates of each point on the unit circle.

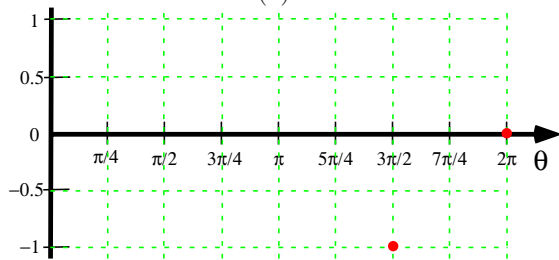
4.4 ♣ **Graphing sine and cosine - (a now-obvious invention from 1730 A.D.)** (Section 1.4.2)

Graph sine and cosine as functions of the angle θ in radians over the range $0 \leq \theta \leq 2\pi$.

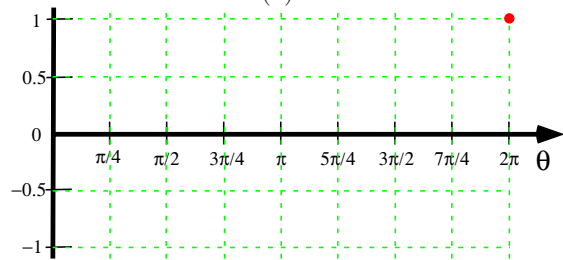
The mathematician was first to regard sine and cosine as **functions** (not just ratios of sides of a triangle).

Result:

$\sin(\theta)$ vs. θ



$\cos(\theta)$ vs. θ



4.5 ♣ **Ranges for arguments and return values for inverse trigonometric functions.**

Determine all real return values and argument values for the following **real** trigonometric and inverse-trigonometric functions in computer languages such as Java, C++, MotionGenesis, and MATLAB®.

Possible return values	Function	Possible argument values	Note
<input type="text"/> ≤ z ≤ <input type="text"/>	$z = \cos(x)$	<input type="text"/> < x < <input type="text"/>	
<input type="text"/> ≤ z ≤ <input type="text"/>	$z = \sin(x)$	<input type="text"/> < x < <input type="text"/>	
<input type="text"/> < z < <input type="text"/>	$z = \tan(x)$	<input type="text"/> < x < <input type="text"/>	$x \neq \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \dots$
<input type="text"/> ≤ z ≤ <input type="text"/>	$z = \text{acos}(x)$	<input type="text"/> ≤ x ≤ <input type="text"/>	
<input type="text"/> ≤ z ≤ <input type="text"/>	$z = \text{asin}(x)$	<input type="text"/> ≤ x ≤ <input type="text"/>	
<input type="text"/> < z < <input type="text"/>	$z = \text{atan}(x)$	<input type="text"/> < x < <input type="text"/>	

4.6 ♣ **What is an angle?** (Section 5.4).

Draw the “geometry equipment” listed in the 1st column of the following table. Complete the 2nd column with appropriate ranges for the angle θ (in degrees).

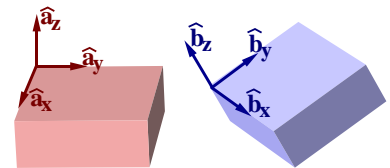


“Geometry equipment”	Draw	Appropriate range for θ
2 lines	<input type="text"/>	$0^\circ \leq \theta \leq$ <input type="text"/>
Vector and line	<input type="text"/>	<input type="text"/> ≤ θ ≤ <input type="text"/>
2 vectors	<input type="text"/>	<input type="text"/> ≤ θ ≤ <input type="text"/>
2 vectors and a sense of positive rotation	<input type="text"/>	<input type="text"/> < θ ≤ <input type="text"/>
2 vectors, a sense of \pm rotation, and time-history/continuity	Not applicable	<input type="text"/> < θ < <input type="text"/>

4.7 **Calculating dot-products, cross-products, and angles between vectors.** (Section 5.2.3).

The ${}^aR^b$ rotation table relates two sets of right-handed, orthogonal, unit vectors, namely $\hat{\mathbf{a}}_x, \hat{\mathbf{a}}_y, \hat{\mathbf{a}}_z$ and $\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$.

${}^aR^b$	$\hat{\mathbf{b}}_x$	$\hat{\mathbf{b}}_y$	$\hat{\mathbf{b}}_z$
$\hat{\mathbf{a}}_x$	0.9623	-0.0842	0.2588
$\hat{\mathbf{a}}_y$	0.1701	0.9284	-0.3304
$\hat{\mathbf{a}}_z$	-0.2125	0.3619	0.9077



- (a) Efficiently determine the following dot-products and angles between vectors (2⁺ significant digits). Then perform the following calculations involving $\vec{\mathbf{v}}_1 = 2\hat{\mathbf{a}}_x$ and $\vec{\mathbf{v}}_2 = \hat{\mathbf{a}}_x + \hat{\mathbf{b}}_x$.

$\hat{\mathbf{a}}_x \cdot \hat{\mathbf{a}}_x =$ <input type="text"/>	$\hat{\mathbf{a}}_y \cdot \hat{\mathbf{a}}_z =$ <input type="text"/>	$\hat{\mathbf{b}}_z \cdot \hat{\mathbf{b}}_y =$ <input type="text"/>
$\hat{\mathbf{a}}_x \cdot \hat{\mathbf{b}}_x =$ <input type="text"/>	$\hat{\mathbf{a}}_x \cdot \hat{\mathbf{b}}_y =$ <input type="text"/>	$\hat{\mathbf{b}}_z \cdot \hat{\mathbf{a}}_y =$ <input type="text"/>
$\angle(\hat{\mathbf{a}}_y, \hat{\mathbf{a}}_y) =$ <input type="text"/> °	$\angle(\hat{\mathbf{b}}_z, \hat{\mathbf{b}}_x) =$ <input type="text"/> °	
$\angle(\hat{\mathbf{a}}_y, \hat{\mathbf{b}}_y) =$ <input type="text"/> °	$\angle(\hat{\mathbf{b}}_y, \hat{\mathbf{a}}_z) =$ <input type="text"/> °	

Result: $\vec{\mathbf{v}}_1 \cdot \vec{\mathbf{v}}_2 =$ $\angle(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2) =$ °
 $\vec{\mathbf{v}}_1 \times \vec{\mathbf{v}}_2 =$ $\hat{\mathbf{b}}_y +$ $\hat{\mathbf{b}}_z =$ $\hat{\mathbf{a}}_y +$ $\hat{\mathbf{a}}_z$

- (b) Express the unit vector $\hat{\mathbf{u}}$ in the direction of $3\hat{\mathbf{a}}_z + 4\hat{\mathbf{b}}_z$ in terms of $\hat{\mathbf{a}}_z$ and $\hat{\mathbf{b}}_z$. Express $\vec{\mathbf{v}} = \hat{\mathbf{a}}_y + \hat{\mathbf{b}}_y$ in terms of $\hat{\mathbf{a}}_x, \hat{\mathbf{a}}_y, \hat{\mathbf{a}}_z$.

Result: $\hat{\mathbf{u}} =$ $\hat{\mathbf{a}}_z +$ $\hat{\mathbf{b}}_z$
 $\vec{\mathbf{v}} =$ $\hat{\mathbf{a}}_x +$ $\hat{\mathbf{a}}_y +$ $\hat{\mathbf{a}}_z$

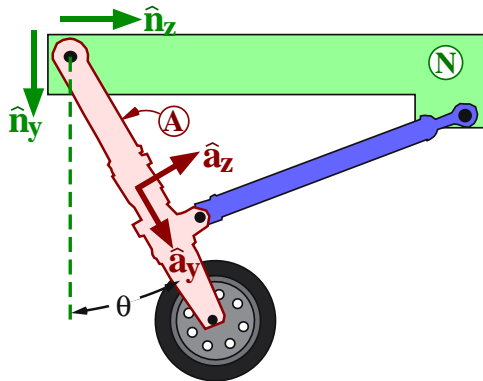
4.8 ♣ **Efficient calculation of the inverse of a rotation matrix.** (Section 5.2.2).

The following rotation matrix R relates two right-handed, orthogonal, unitary bases. Calculate its inverse by-hand (no calculator) in less than 30 seconds.

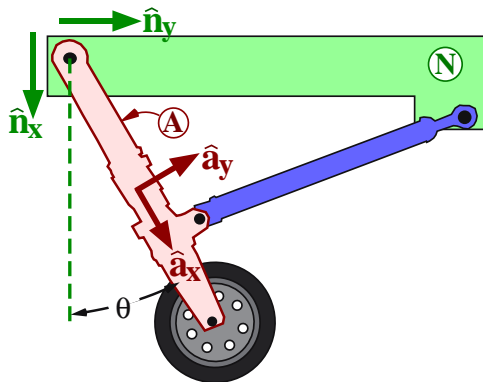
$$R = \begin{bmatrix} 0.3830 & -0.6634 & 0.6428 \\ 0.9237 & 0.2795 & -0.2620 \\ -0.0058 & 0.6941 & 0.7198 \end{bmatrix} \Rightarrow R^{-1} = \begin{bmatrix} \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} \end{bmatrix}$$

4.9 **SohCahToa: Rotation tables for a landing gear system.** (Section 5.3).

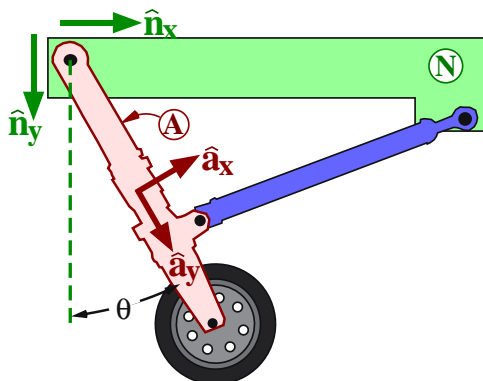
The figures below show three versions of the same landing gear system with a strut A that has a simple rotation relative to a fuselage N . Each figure has a different orientation for right-handed orthogonal unit vectors $\hat{n}_x, \hat{n}_y, \hat{n}_z$ (fixed in N) and $\hat{a}_x, \hat{a}_y, \hat{a}_z$ (fixed in A). **Redraw** \hat{n}_y, \hat{n}_z and $\hat{a}_x, \hat{a}_y, \hat{a}_z$ so it is **easy to see** sines and cosines. Then, form the ${}^aR^n$ rotation table for each figure.¹



${}^aR^n$	\hat{n}_x	\hat{n}_y	\hat{n}_z
\hat{a}_x	1	<input type="text"/>	<input type="text"/>
\hat{a}_y	0	$\cos(\theta)$	$\sin(\theta)$
\hat{a}_z	0	$-\sin(\theta)$	$\cos(\theta)$



${}^aR^n$	\hat{n}_x	\hat{n}_y	\hat{n}_z
\hat{a}_x	<input type="text"/>	<input type="text"/>	<input type="text"/>
\hat{a}_y	<input type="text"/>	<input type="text"/>	<input type="text"/>
\hat{a}_z	<input type="text"/>	<input type="text"/>	<input type="text"/>



${}^aR^n$	\hat{n}_x	\hat{n}_y	\hat{n}_z
\hat{a}_x	<input type="text"/>	<input type="text"/>	<input type="text"/>
\hat{a}_y	<input type="text"/>	<input type="text"/>	<input type="text"/>
\hat{a}_z	<input type="text"/>	<input type="text"/>	<input type="text"/>

¹Each figure has two missing vectors (e.g., \hat{n}_x and \hat{a}_x are missing from the first figure). Use the fact that each set of vectors is **right-handed** to add the missing vectors to each figure.