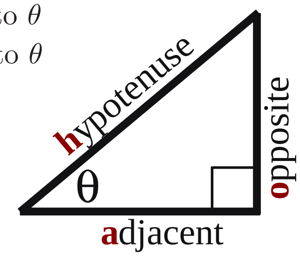
4.1 ♣ SohCahToa: Sine, cosine, tangent as ratios of sides of a right triangle. (Section 1.4)

The following shows a ***right triangle*** with one of its angles labeled as θ.
Write definitions for sine, cosine, and tangent in terms of:

- **h**ypotenuse – the triangle's longest side (opposite the 90° angle).
- **o**pposite – the side opposite to θ
- **a**djacent – the side adjacent to θ

Note: A mnemonic for these definitions is "***SohCahToa***".

Note: A ***right triangle*** is a triangle with a 90° angle.



$\sin(\theta) \triangleq \dfrac{\quad}{\quad}$

$\cos(\theta) \triangleq \dfrac{\quad}{\quad}$

$\tan(\theta) \triangleq \dfrac{\quad}{\quad} = \dfrac{\sin(\theta)}{\cos(\theta)}$

4.2 ♣ Pythagorean theorem and law of cosines - memorize. (Section 1.4.1).

Draw a right-triangle with a hypotenuse of length c and other sides of length a and b. Relate c to a and b with the ***Pythagorean theorem***.

Result: $c^2 = \quad$

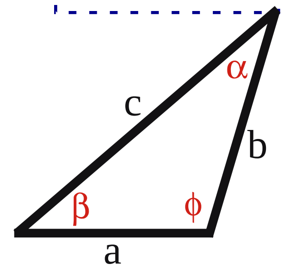
A non-right-triangle has angles α, β, ϕ opposite sides a, b, c, respectively. Use the ***law of cosines*** to complete each formula below.

Result:

$c^2 = a^2 + b^2 - 2\,a\,b\cos(\phi)$

$a^2 = \quad$

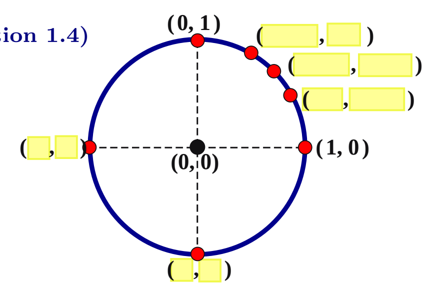
$b^2 = \quad$



The ***Pythagorean theorem*** is a special case of the ***law of cosines***. **True/False**. (circle one).

4.3 ♣ Memorize common sines and cosine. (Section 1.4)

$\sin(0°) =$	$\cos(0°) =$
$\sin(30°) =$	$\cos(30°) =$
$\sin(45°) =$	$\cos(45°) =$
$\sin(60°) =$	$\cos(60°) =$
$\sin(90°) =$	$\cos(90°) =$



Label the coordinates of each point on the unit circle.

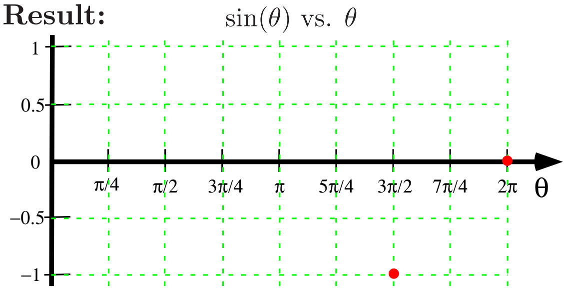
4.4 ♣ Graphing sine and cosine - (a now-obvious invention from 1730 A.D.) (Section 1.4.2)

Graph sine and cosine as functions of the angle θ in radians over the range $0 \le \theta \le 2\,\pi$.

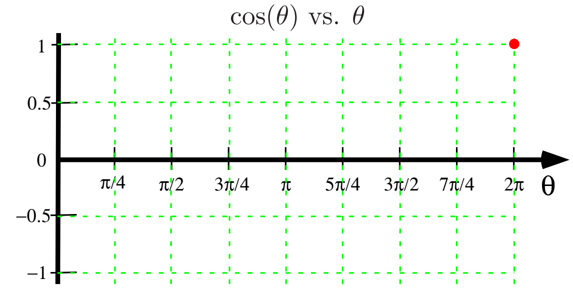
The mathematician ______ was first to regard sine and cosine as **functions** (not just ratios of sides of a triangle).

Result:

$\sin(\theta)$ vs. θ



$\cos(\theta)$ vs. θ



4.5 ♣ Ranges for arguments and return values for inverse trigonometric functions.

Determine all real return values and argument values for the following <u>real</u> trigonometric and inverse-trigonometric functions in computer languages such as Java, C^{++}, **M**otion**G**enesis, and MATLAB®.

Possible return values	Function	Possible argument values	Note
$\le z \le$	$z = \cos(x)$	$< x <$	
$\le z \le$	$z = \sin(x)$	$< x <$	
$-\infty < z < \infty$	$z = \tan(x)$	$-\infty < x < \infty$	$x \ne \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$
$\le z \le$	$z = \text{acos}(x)$	$\le x \le$	
$\le z \le$	$z = \text{asin}(x)$	$\le x \le$	
$-\pi/2 < z < \pi/2$	$z = \text{atan}(x)$	$-\infty < x < \infty$	
$\le z \le$	$z = \texttt{atan2}(y, x)$	$< y <$ $< x <$	$\texttt{atan2}(0, 0)$ is undefined

4.6 ♣ What is an angle? (Section 5.6).

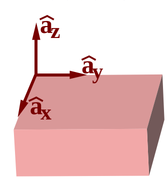
<u>Draw</u> the "geometry equipment" listed in the 1st column of the following table. Complete the 2nd column with appropriate ranges for the angle θ (in degrees).

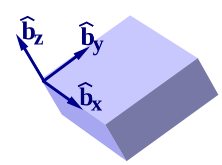
"Geometry equipment"	Draw	Appropriate range for θ
2 lines		$0° \le \theta \le$
Vector and line		$\le \theta \le$
2 vectors		$\le \theta \le$
2 vectors and a sense of positive rotation		$< \theta \le$
2 vectors, a sense of $\pm$ rotation, and time-history/continuity	Not applicable	$< \theta <$

4.7 Calculating dot-products, cross-products, and angles between vectors. (Section 5.4.3).

The ${}^aR^b$ rotation table relates two sets of right-handed, orthogonal, unit vectors, namely $\hat{\mathbf{a}}_x, \hat{\mathbf{a}}_y, \hat{\mathbf{a}}_z$ and $\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$.

${}^aR^b$	$\hat{\mathbf{b}}_x$	$\hat{\mathbf{b}}_y$	$\hat{\mathbf{b}}_z$
$\hat{\mathbf{a}}_x$	0.9623	−0.0842	0.2588
$\hat{\mathbf{a}}_y$	0.1701	0.9284	−0.3304
$\hat{\mathbf{a}}_z$	−0.2125	0.3619	0.9077





(a) Efficiently determine the following dot-products and angles between vectors (2$^+$ significant digits). Then perform the following calculations involving $\vec{\mathbf{v}}_1 = 2\,\hat{\mathbf{a}}_x$ and $\vec{\mathbf{v}}_2 = \hat{\mathbf{a}}_x + \hat{\mathbf{b}}_x$.

$\hat{\mathbf{a}}_x \cdot \hat{\mathbf{a}}_x =$	$\hat{\mathbf{a}}_y \cdot \hat{\mathbf{a}}_z =$	$\hat{\mathbf{b}}_z \cdot \hat{\mathbf{b}}_y =$
$\hat{\mathbf{a}}_x \cdot \hat{\mathbf{b}}_x =$	$\hat{\mathbf{a}}_x \cdot \hat{\mathbf{b}}_y =$	$\hat{\mathbf{b}}_z \cdot \hat{\mathbf{a}}_y =$
$\angle(\hat{\mathbf{a}}_y, \hat{\mathbf{a}}_y) =$ °	$\angle(\hat{\mathbf{b}}_z, \hat{\mathbf{b}}_x) =$ °	
$\angle(\hat{\mathbf{a}}_y, \hat{\mathbf{b}}_y) =$ °	$\angle(\hat{\mathbf{b}}_y, \hat{\mathbf{a}}_z) =$ °	

Result: $\vec{\mathbf{v}}_1 \cdot \vec{\mathbf{v}}_2 =$ $\qquad$ $\angle(\vec{\mathbf{v}}_1, \vec{\mathbf{v}}_2) =$ °

$\vec{\mathbf{v}}_1 \times \vec{\mathbf{v}}_2 = \quad \hat{\mathbf{b}}_y + \quad \hat{\mathbf{b}}_z = \quad \hat{\mathbf{a}}_y + \quad \hat{\mathbf{a}}_z$

(b) Express the unit vector $\hat{\mathbf{u}}$ in the direction of $3\,\hat{\mathbf{a}}_z + 4\,\hat{\mathbf{b}}_z$ in terms of $\hat{\mathbf{a}}_z$ and $\hat{\mathbf{b}}_z$. Express $\vec{\mathbf{v}} = \hat{\mathbf{a}}_y + \hat{\mathbf{b}}_y$ in terms of $\hat{\mathbf{a}}_x, \hat{\mathbf{a}}_y, \hat{\mathbf{a}}_z$.

Result: $\hat{\mathbf{u}} = \quad \hat{\mathbf{a}}_z + \quad \hat{\mathbf{b}}_z$

$\vec{\mathbf{v}} = \quad \hat{\mathbf{a}}_x + \quad \hat{\mathbf{a}}_y + \quad \hat{\mathbf{a}}_z$

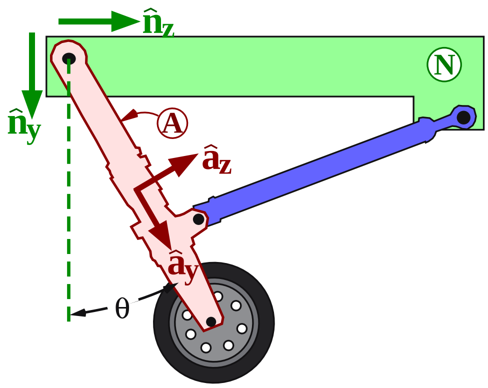
4.8 ♣ Efficient calculation of the inverse of a rotation matrix. (Section 5.4.2).

The following rotation matrix R relates two right-handed, orthogonal, unitary bases. Calculate its inverse by-hand (no calculator) in less than 30 seconds.

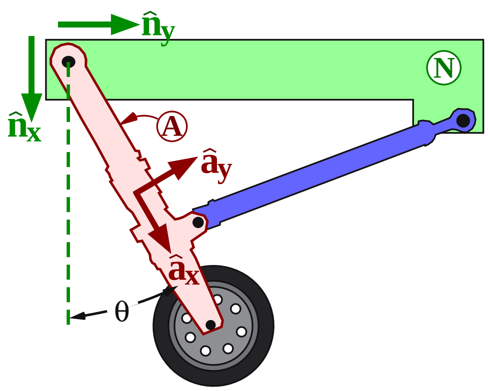
$$R \;=\; \begin{bmatrix} 0.3830 & -0.6634 & 0.6428 \\ 0.9237 & 0.2795 & -0.2620 \\ -0.0058 & 0.6941 & 0.7198 \end{bmatrix} \qquad \Rightarrow \qquad R^{-1} \;=\; \begin{bmatrix} \; & \; & \; \\ \; & \; & \; \\ \; & \; & \; \end{bmatrix}$$

4.9 SohCahToa: Rotation tables for a landing gear system. (Section 5.5).

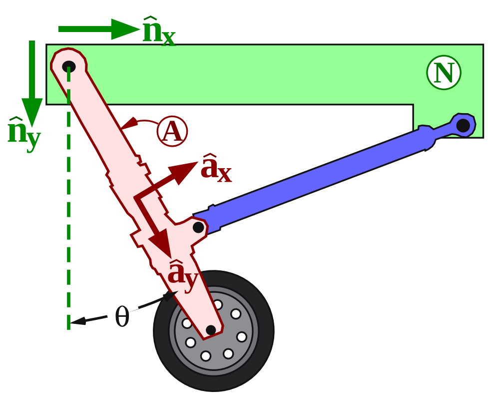
The figures below show three versions of the same landing gear system with a strut A that has a simple rotation relative to a fuselage N. Each figure has a different orientation for right-handed orthogonal unit vectors $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ (fixed in N) and $\widehat{\mathbf{a}}_x$, $\widehat{\mathbf{a}}_y$, $\widehat{\mathbf{a}}_z$ (fixed in A). **Redraw** $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ and $\widehat{\mathbf{a}}_x$, $\widehat{\mathbf{a}}_y$, $\widehat{\mathbf{a}}_z$ so it is **easy to see** sines and cosines. Then, form the ${}^aR^n$ rotation table for each figure.[1]



${}^aR^n$	$\widehat{\mathbf{n}}_x$	$\widehat{\mathbf{n}}_y$	$\widehat{\mathbf{n}}_z$
$\widehat{\mathbf{a}}_x$	1		
$\widehat{\mathbf{a}}_y$	0	$\cos(\theta)$	$\sin(\theta)$
$\widehat{\mathbf{a}}_z$	0	$-\sin(\theta)$	$\cos(\theta)$



${}^aR^n$	$\widehat{\mathbf{n}}_x$	$\widehat{\mathbf{n}}_y$	$\widehat{\mathbf{n}}_z$
$\widehat{\mathbf{a}}_x$			
$\widehat{\mathbf{a}}_y$			
$\widehat{\mathbf{a}}_z$			



${}^aR^n$	$\widehat{\mathbf{n}}_x$	$\widehat{\mathbf{n}}_y$	$\widehat{\mathbf{n}}_z$
$\widehat{\mathbf{a}}_x$			
$\widehat{\mathbf{a}}_y$			
$\widehat{\mathbf{a}}_z$			

[1] Each figure has two missing vectors (e.g., $\widehat{\mathbf{n}}_x$ and $\widehat{\mathbf{a}}_x$ are missing from the first figure). Use the fact that each set of vectors is **right-handed** to add the missing vectors to each figure.