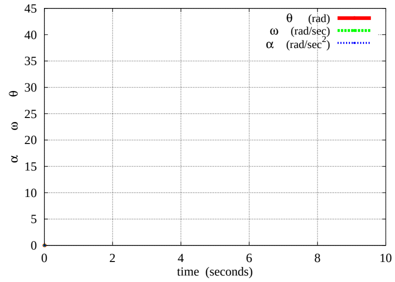
6.1 FE/EIT Review – Motion graph: $\mathbf{T} \Rightarrow \alpha \Rightarrow \omega \Rightarrow \theta$

A wind turbine generates electricity from wind forces that create a time-dependent torque modeled as $\mathrm{T} = 20 \, \frac{\mathrm{N\,m}}{\mathrm{sec}} * t$. Measures of the wind turbine's angular acceleration α, angular velocity ω, and angle θ are related as shown below, where $\mathrm{I} = 80 \text{ kg m}^2$ is the relevant moment of inertia.

$$\mathrm{T} \underset{(\mathbf{2D})}{=} \mathrm{I}\,\alpha \qquad \alpha = \frac{d\omega}{dt} \qquad \omega \underset{(\mathbf{2D})}{=} \frac{d\theta}{dt}$$

Graph α in $\frac{\mathrm{rad}}{\mathrm{sec}^2}$, ω in $\frac{\mathrm{rad}}{\mathrm{sec}}$, and θ in rad for $0 \le t \le 10$ sec. Use initial values (i.e, values at $t = 0$) of $\omega = 0$ and $\theta = 0$.



6.2 Drawing a reference frame and unit vector bases. (Section 7.2)

Draw a reference frame or rigid body B, shaped like a uniform-density doughnut (having a hole).
Draw a right-handed orthogonal bases fixed in B having unit vectors $\widehat{\mathbf{b}}_{\mathrm{x}}$, $\widehat{\mathbf{b}}_{\mathrm{y}}$, $\widehat{\mathbf{b}}_{\mathrm{z}}$.
Draw a different right-handed orthogonal bases fixed in B with unit vectors $\widehat{\mathbf{b}}_1$, $\widehat{\mathbf{b}}_2$, $\widehat{\mathbf{b}}_3$.
Draw a properly located center of mass symbol ◐ and label this point as B_{cm}.
Draw a point B_{o} fixed on B, at a location different than B_{cm}.

6.3 ♣ Words and pictures for ${}^{b}R^{a}$, ${}^{N}\vec{\boldsymbol{\omega}}^{B}$, ${}^{N}\vec{\boldsymbol{\alpha}}^{B}$. (Chapters 4 and 7)

${}^{b}R^{a}$ – Description (**words**)	${}^{N}\vec{\boldsymbol{\omega}}^{B}$ – Description (**words**)	${}^{N}\vec{\boldsymbol{\alpha}}^{B}$ – Description (**words**)
Draw b and a	**Draw** B and N	

6.4 ♣ Definitions of angular velocity. (Section 7.3.3).

The definition of angular velocity of $\vec{\boldsymbol{\omega}} \triangleq \dot{\theta}\,\vec{\mathbf{k}}$ is a functional operational definition, i.e., in general, it is useful for calculating angular velocity and proving its properties (2D or 3D). **True/False**

6.5 ♣ Textbook and Internet definitions of 3D angular velocity (Section 7.3).

Famed dynamicist Thomas Kane called angular velocity "**one of the most misunderstood concepts in kinematics**." Report two definitions of angular velocity and determine if the quantities appearing in the definition are **rigorously defined** - and whether they are generally applicable for 3D kinematics or only apply for simple angular velocity (described in Section 7.3.3).

Note: A definition should be able to **prove** important theorems [such as the angular velocity addition theorem of equation (7.4) and the golden rule for vector differentiation in equation (7.1)] and allow for angular velocity **calculations**.

	Source	Definition	Rigorously defined	Works for 3D kinematics?
Textbook:	List textbook	Record equation/definition	**Yes/No**	**Yes/No**
Internet:	Provide .html	Record equation/definition	**Yes/No**	**Yes/No**

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6.6 ♣ Concept: What objects have a unique angular velocity/acceleration? (Sections 7.3, 7.4).

${}^N\vec{\boldsymbol{\omega}}^S$, the angular velocity of an object S in a reference frame N is to be determined.
In general and **without ambiguity**, S could be a (circle ***all*** appropriate objects):

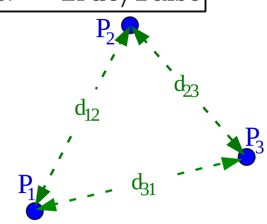
Real number	Point	Reference Frame	Mass center of a set of particles
Vector	Set of Points	Rigid Body	Mass center of a rigid body
Matrix	Particle	Flexible Body	Set of flexible bodies
3D Orthogonal unit basis	Set of Particles	Set of Rigid bodies	System of particles and bodies

Repeat for ${}^N\vec{\boldsymbol{\alpha}}^S$, the angular acceleration of an object S in a reference frame N box appropriate objects.

6.7 ♣ What is a reference frame, rigid body, and orthogonal basis? (Sections 3.1 and 7.2)

#	Statement (regard "***rigid body***" as a massive 2D or 3D rigid object)	True or False
a	A reference frame has all the attributes of a rigid body.	**True/False**
b	A rigid body has all the attributes of a reference frame.	**True/False**
c	A reference frame with time-invariant distributed mass is a rigid body.	**True/False**
d	A massless rigid object is a reference frame.	**True/False**
e	The definition of a reference frame implies a sense of time.	**True/False**
f	A rigid body B may have an angular velocity in a reference frame N.	**True/False**
g	A point Q has a uniquely-defined angular velocity in a reference frame N.	**True/False**
h	The reference frame B implies unique orthogonal unit vectors $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$.	**True/False**
i	The right-handed orthogonal unit vectors $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$ imply a unique reference frame.	**True/False**
j	The reference frame B implies a unique rigid frame.	**True/False**
k	A rigid frame with origin B_o and basis $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$ implies a unique reference frame.	**True/False**

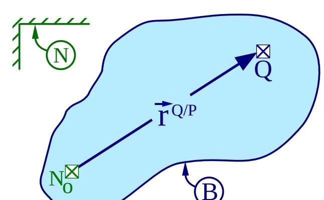
6.8 ♣ Concept: Reference frames and vector bases. (Sections 3.1 and 7.2)



Consider 3 distinct non-collinear points P_1, P_2, P_3 and the non-zero distances d_{12}, d_{23}, d_{31} between them. In general, determine if each object below can **always** be constructed from P_1, P_2, P_3 under the listed condition.
For each "**Yes**" answer, **draw** the object.

Condition	Object to be constructed	Object can be constructed?	If Yes, **Draw**
d_{12}, d_{23}, d_{31} are **constant**	Vector basis that spans 3D space	**Yes/No**	
d_{12}, d_{23}, d_{31} are **variable**	Vector basis that spans 3D space	**Yes/No**	
d_{12}, d_{23}, d_{31} are **constant**	Right-handed, orthogonal, unitary basis	**Yes/No**	
d_{12}, d_{23}, d_{31} are **variable**	Right-handed, orthogonal, unitary basis	**Yes/No**	
d_{12}, d_{23}, d_{31} are **constant**	Unique reference frame	**Yes/No**	
d_{12}, d_{23}, d_{31} are **variable**	Unique reference frame	**Yes/No**	

6.9 ♣ Vector differentiation concepts "$v = \omega\, r$". (Section 7.3).



Point Q is fixed on a rigid body B. Point N_o is fixed in a reference frame N and does not move on B. Complete the following proof that shows how $\vec{\mathbf{v}}$ (Q's velocity in N) can be written in terms of ${}^N\vec{\boldsymbol{\omega}}^B$ (B's angular velocity in N) and $\vec{\mathbf{r}}$ (Q's position vector from N_o).

Mathematical statement	Reasoning (explain each step in the proof with a brief phrase)
$\vec{\mathbf{v}} \triangleq \frac{{}^N d\,\vec{\mathbf{r}}}{dt}$	Definition of Q's velocity in N
$= \square + \square$	
$= \vec{\mathbf{0}} + {}^N\vec{\boldsymbol{\omega}}^B \times \vec{\mathbf{r}}$	

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6.10 ♣ Concepts: What objects have a uniquely-defined angular velocity? (Section 7.3).

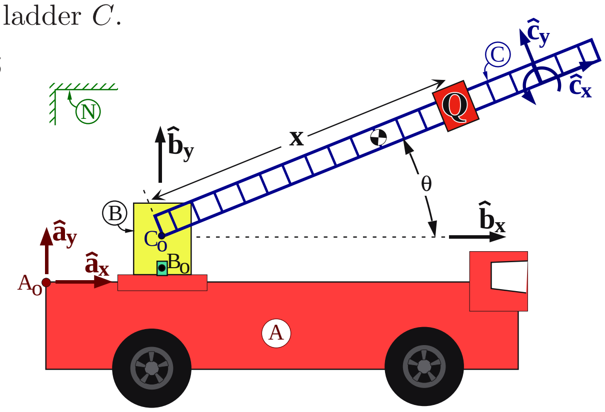
#	For: ${}^{A}\vec{\boldsymbol{\omega}}^{B}$ (B's angular velocity in A)	Object B		Object A	
a.	It is possible to find the angular velocity of a	point	in a	reference frame.	**True/False**
b.	It is possible to find the angular velocity of a	rigid body	in a	particle.	**True/False**
c.	It is possible to find the angular velocity of a	rigid body	in a	reference frame.	**True/False**
d.	It is possible to find the angular velocity of a	reference frame	in a	rigid body.	**True/False**
e.	It is possible to find the angular velocity of a	reference frame	in a	flexible body.	**True/False**
f.	It is possible to find the angular velocity of a	flexible body	in a	reference frame.	**True/False**

6.11 ♣ Rotational kinematics of a fire ladder. (Sections 7.3.3, 7.3.5, 7.3.6).

The following figure shows a fire truck chassis A traveling at constant speed in straight-line motion on Earth (A does not rotate relative to Earth). Earth is a ***Newtonian reference frame*** N.
A rigid hub B is connected to fire truck A by a revolute motor at point B_o of B.
A rigid ladder C is connected to hub B by a revolute motor at point C_o of C .
A fire-fighter Q (modeled as a particle of mass m) climbs ladder C.

Right-handed orthogonal unit vectors $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$; $\hat{\mathbf{b}}_x$, $\hat{\mathbf{b}}_y$, $\hat{\mathbf{b}}_z$; $\hat{\mathbf{c}}_x$, $\hat{\mathbf{c}}_y$, $\hat{\mathbf{c}}_z$; are fixed in A, B, C, with:

- $\hat{\mathbf{a}}_x$ pointing forward on the fire truck
- $\hat{\mathbf{a}}_y$ vertically-upward and from B_o to C_o
- $\hat{\mathbf{b}}_y = \hat{\mathbf{a}}_y$ parallel to the axis of the revolute motor that connects B and A
- $\hat{\mathbf{b}}_z = \hat{\mathbf{c}}_z$ parallel to the axis of the revolute motor that connects B and C
- $\hat{\mathbf{c}}_x$ directed from C_o to Q (along C's long axis)



Note: **Visualize** C's "**Body yz**" (or "**Space zy**") rotation sequence in N (e.g., with a ruler).

Quantity	Symbol	Type
$\hat{\mathbf{b}}_y$ measure of B's angular velocity in A	ω_B	Constant
Angle from $\hat{\mathbf{b}}_x$ to $\hat{\mathbf{c}}_x$ with $^{+}\hat{\mathbf{c}}_z$ sense	θ	Variable

${}^{c}R^{b}$	___	___	___
___	___	___	___
___	___	___	___
___	___	___	___

(a) Complete the previous ${}^{c}R^{b}$ rotation table (to the right).
Note: ${}^{c}R^{b}$ is unnecessary for the remainder of this problem.

(b) Clarify the process to determine ${}^{B}\vec{\boldsymbol{\omega}}^{C}$, then express it in terms of $\hat{\mathbf{b}}_x$, $\hat{\mathbf{b}}_y$, $\hat{\mathbf{b}}_z$. (Section 7.3.3).

- C's angular velocity in B is **simple** since ___ is fixed in **both** ___ and ___.
- ___ is the time-derivative of the angle between ___ and ___.
- The sign (±) was determined using the ___-hand rule (sweep from ___ to ___).
- ${}^{B}\vec{\boldsymbol{\omega}}^{C}$ = ___

(c) B's angular velocity in A is known to be a **simple** angular velocity of ${}^{A}\vec{\boldsymbol{\omega}}^{B} = \omega_B\,\hat{\mathbf{b}}_y$ because $\hat{\mathbf{b}}_y$ is a vector fixed in **both** ___ and ___.

(d) Form C's angular velocity in N and express it in terms of $\hat{\mathbf{b}}_x$, $\hat{\mathbf{b}}_y$, $\hat{\mathbf{b}}_z$.
Result:

$$ {}^{N}\vec{\boldsymbol{\omega}}^{C} \underset{(7.4)}{=} \_\_\vec{\boldsymbol{\omega}}\_\_ + \_\_\vec{\boldsymbol{\omega}}\_\_ + \_\_\vec{\boldsymbol{\omega}}\_\_ = \vec{\mathbf{0}} + \_\_\,\hat{\mathbf{b}}_y + \_\_\,\hat{\mathbf{b}}_z $$

(e) When both ω_B and $\dot{\theta}$ are ***constant***, ${}^{N}\vec{\boldsymbol{\alpha}}^{C} = \vec{\mathbf{0}}$. **True/False**.

(f) Write the definition for C's angular acceleration in N and form ${}^{N}\vec{\boldsymbol{\alpha}}^{C}$. (Sections 7.4, 7.3).
Result:

$$ {}^{N}\vec{\boldsymbol{\alpha}}^{C} \underset{(7.8)}{\triangleq} \_\_\_ \qquad {}^{N}\vec{\boldsymbol{\alpha}}^{C} \underset{(7.1)}{=} \omega_B\,\dot{\theta}\,\hat{\mathbf{b}}_x + \ddot{\theta}\,\hat{\mathbf{b}}_z $$

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