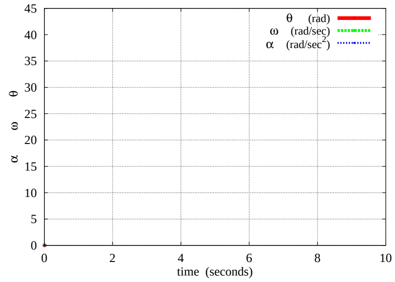
6.1 FE/EIT Review – Motion graph: $\boxed{\mathrm{T} \;\Rightarrow\; \alpha \;\Rightarrow\; \omega \;\Rightarrow\; \theta}$

A wind turbine generates electricity from wind forces that create a time-dependent torque modeled as $\mathrm{T} = 20 \, \frac{\mathrm{N\,m}}{\mathrm{sec}} * t$. Measures of the wind turbine's angular acceleration α, angular velocity ω, and angle θ are related as shown below, where $\mathrm{I} = 80 \text{ kg m}^2$ is the relevant moment of inertia.

$$\boxed{\mathrm{T} \underset{(\mathbf{2D})}{=} \mathrm{I}\,\alpha \qquad \alpha = \frac{d\omega}{dt} \qquad \omega \underset{(\mathbf{2D})}{=} \frac{d\theta}{dt}}$$

Graph α in $\frac{\mathrm{rad}}{\mathrm{sec}^2}$, ω in $\frac{\mathrm{rad}}{\mathrm{sec}}$, and θ in rad for $0 \le t \le 10$ sec. Use initial values (i.e, values at $t = 0$) of $\omega = 0$ and $\theta = 0$.



6.2 Drawing a reference frame and unit vector bases. (Section 7.2)

Draw a rigid body B, shaped like a uniform-density doughnut (having a hole).
Draw a right-handed orthogonal bases fixed in B having unit vectors $\widehat{\mathbf{b}}_{\mathrm{x}}$, $\widehat{\mathbf{b}}_{\mathrm{y}}$, $\widehat{\mathbf{b}}_{\mathrm{z}}$.
Draw a different right-handed orthogonal bases fixed in B with unit vectors $\widehat{\mathbf{b}}_1$, $\widehat{\mathbf{b}}_2$, $\widehat{\mathbf{b}}_3$.
Draw a properly located center of mass symbol ◐ and label this point as B_{cm}.
Draw a point B_{o} fixed on B (called B's origin) at a location different than B_{cm}.

6.3 ♣ Words and pictures for ${}^{b}R^{a}$, ${}^{N}\vec{\boldsymbol{\omega}}^{B}$, ${}^{N}\vec{\boldsymbol{\alpha}}^{B}$. (Chapters 5 and 7)

${}^{b}R^{a}$ – Description (**words**)	${}^{N}\vec{\boldsymbol{\omega}}^{B}$ – Description (**words**)	${}^{N}\vec{\boldsymbol{\alpha}}^{B}$ – Description (**words**)
Draw b and a	**Draw** B and N	

6.4 ♣ Definition of angular velocity? (Section 7.3.3).

The definition of angular velocity of $\vec{\boldsymbol{\omega}} \triangleq \dot{\theta}\,\vec{\mathbf{k}}$ is generally useful for calculating angular velocity and proving its properties (for both 2D and/or 3Danalysis). **True/False**

6.5 ♣ Optional: Textbook/Internet definitions of 3D angular velocity (Section 7.3).

Famed dynamicist Thomas Kane called angular velocity "**one of the most misunderstood concepts in kinematics.**" Report two definitions of angular velocity and determine if the quantities appearing in the definition are **rigorously defined** - and whether they are generally applicable for 3D kinematics or only apply for simple angular velocity (described in Section 7.3.3).

Note: A definition should be able to **prove** important theorems [such as the angular velocity addition theorem of equation (7.4) and the golden rule for vector differentiation in equation (7.1)] and allow for angular velocity **calculations**.

Source (reference)	Definition	Rigorously defined	Works for 3D kinematics?
List textbook or .html link	Record equation/definition	**Yes/No**	**Yes/No**
List textbook or .html link	Record equation/definition	**Yes/No**	**Yes/No**

6.6 Angular velocity of a Ferris-wheel seat (courtesy of David Levinson).

Given: The rigid seat on a Ferris wheel does not change its orientation relative to ground as the Ferris wheel rotates.
Decide: The seat's angular velocity $\vec{\boldsymbol{\omega}}$ relative to ground (circle one):

Is zero $\vec{\boldsymbol{\omega}} = \vec{\mathbf{0}}$	Is constant $\vec{\boldsymbol{\omega}} = \vec{\textbf{Constant}} \neq \vec{\mathbf{0}}$	Varies $\vec{\boldsymbol{\omega}} = \vec{\boldsymbol{\omega}}(t)$	Does not exist

6.7 ♣ Concept: What objects have a unique angular velocity/acceleration? (Sections 7.3, 7.4).
The angular velocity $\vec{\boldsymbol{\omega}}$ of some object S relative to Earth is to be determined.
This object S could be a (circle ***all*** objects that have an ***unambiguously*** defined angular velocity $\vec{\boldsymbol{\omega}}$):

Real number	Matrix	Set of points	Mass center of a rigid body
Vector	Point	Reference frame	Flexible body
3D orthogonal unit basis	Particle	3D rigid body	System of particles and bodies

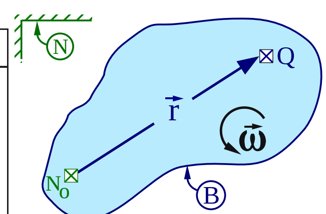
Repeat for the angular acceleration $\vec{\boldsymbol{\alpha}}$ of some object S relative to Earth $\boxed{\text{box appropriate objects}}$.

6.8 ♣ Concepts: What objects have a uniquely-defined angular velocity? (Section 7.3).

a. It is possible to find the angular velocity of a	point	in a	reference frame.	**True/False**
b. It is possible to find the angular velocity of a	3D rigid body	in a	particle.	**True/False**
c. It is possible to find the angular velocity of a	3D rigid body	in a	reference frame.	**True/False**
d. It is possible to find the angular velocity of a	reference frame	in a	3D rigid body.	**True/False**
d. It is possible to find the angular velocity of a	3D rigid basis	in a	reference frame.	**True/False**
e. It is possible to find the angular velocity of a	reference frame	in a	flexible body.	**True/False**
f. It is possible to find the angular velocity of a	flexible body	in a	reference frame.	**True/False**

6.9 ♣ Vector differentiation concepts "$v = \omega\, r$". (Section 7.3).
Point Q is fixed on a rigid body B. Point N_o is fixed in a reference frame N and does not move on B. Complete the following proof that shows how $\vec{\mathbf{v}}$ (Q's velocity in N) can be written in terms of ${}^N\vec{\boldsymbol{\omega}}^B$ (B's angular velocity in N) and $\vec{\mathbf{r}}$ (Q's position vector from N_o).

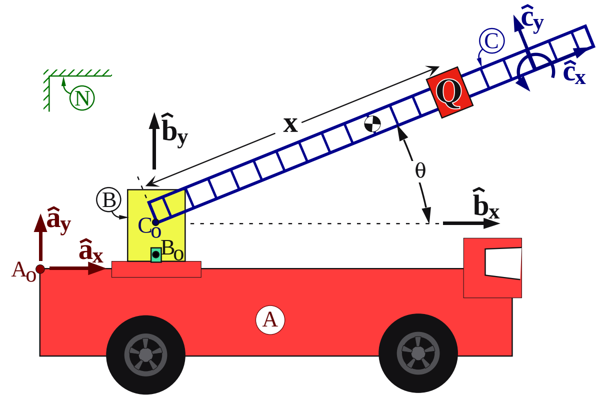
Mathematical statement	Reasoning (explain each step below with a brief phrase)
$\vec{\mathbf{v}} \triangleq \frac{{}^N d\,\vec{\mathbf{r}}}{dt}$	Definition of Q's velocity in N
$= \square + \square$	
$= \vec{\mathbf{0}} + {}^N\vec{\boldsymbol{\omega}}^B \times \vec{\mathbf{r}}$	



6.10 ♣ Rotational kinematics of a fire ladder. (Sections 7.3.3, 7.3.5, 7.3.6).
The following figure shows a fire truck chassis A traveling at constant speed in straight-line motion on Earth (A does not rotate relative to Earth). Earth is a ***Newtonian reference frame*** N.
A rigid hub B is connected to fire truck A by a revolute motor at point B_o of B.
A rigid ladder C is connected to hub B by a revolute motor at point C_o of C .
A fire-fighter Q (modeled as a particle of mass m) climbs ladder C.

Right-handed orthogonal unit vectors $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$; $\hat{\mathbf{b}}_x$, $\hat{\mathbf{b}}_y$, $\hat{\mathbf{b}}_z$; $\hat{\mathbf{c}}_x$, $\hat{\mathbf{c}}_y$, $\hat{\mathbf{c}}_z$; are fixed in A, B, C, with:

- $\hat{\mathbf{a}}_x$ pointing forward on the fire truck
- $\hat{\mathbf{a}}_y$ vertically-upward and from B_o to C_o
- $\hat{\mathbf{b}}_y = \hat{\mathbf{a}}_y$ parallel to the axis of the revolute motor that connects B and A
- $\hat{\mathbf{b}}_z = \hat{\mathbf{c}}_z$ parallel to the axis of the revolute motor that connects B and C
- $\hat{\mathbf{c}}_x$ directed from C_o to Q (along C's long axis)



Note: **Visualize** C's "**Body yz**" (or "**Space zy**") rotation sequence in N (e.g., with a ruler).

Quantity	Symbol	Type
$\hat{\mathbf{b}}_y$ measure of B's angular velocity in A	ω_B	Constant
Angle from $\hat{\mathbf{b}}_x$ to $\hat{\mathbf{c}}_x$ with $^{+}\hat{\mathbf{c}}_z$ sense	θ	Variable

${}^{c}R^{b}$	____	____	____
____	____	____	____
____	____	____	____
____	____	____	____

(a) Complete the previous ${}^{c}R^{b}$ rotation table (to the right).
Note: ${}^{c}R^{b}$ is unnecessary for the remainder of this problem.

(b) Clarify the process to determine C's angular velocity in B. (Section 7.3.3).

- ${}^{B}\vec{\boldsymbol{\omega}}^{C}$ is a **simple** angular velocity because $\hat{\mathbf{b}}_z$ is a vector fixed in **both** ____ and ____.
- ${}^{B}\vec{\boldsymbol{\omega}}^{C} =$ ____, where ____ is the angle between ____ and ____.
- The sign ($\pm$) is determined using the ____-hand rule (sweep from ____ to ____).

(c) ${}^{A}\vec{\boldsymbol{\omega}}^{B} = \omega_B\,\hat{\mathbf{b}}_y$ is a **simple** angular velocity because $\hat{\mathbf{b}}_y$ is a vector fixed in **both** ____ and ____.

(d) Form C's angular velocity in N and express it in terms of $\hat{\mathbf{b}}_x$, $\hat{\mathbf{b}}_y$, $\hat{\mathbf{b}}_z$.
Result: $${}^{N}\vec{\boldsymbol{\omega}}^{C} \underset{(7.4)}{=} \text{\_\_}\vec{\boldsymbol{\omega}}\text{\_\_} + \text{\_\_}\vec{\boldsymbol{\omega}}\text{\_\_} + \text{\_\_}\vec{\boldsymbol{\omega}}\text{\_\_} = \vec{\mathbf{0}} + \text{\_\_}\,\hat{\mathbf{b}}_y + \text{\_\_}\,\hat{\mathbf{b}}_z$$

(e) When both ω_B and $\dot{\theta}$ are ***constant***, ${}^{N}\vec{\boldsymbol{\alpha}}^{C} = \vec{\mathbf{0}}$. **True/False**.

(f) Write the definition for C's angular acceleration in N and form ${}^{N}\vec{\boldsymbol{\alpha}}^{C}$. (Sections 7.4, 7.3).
Result: $${}^{N}\vec{\boldsymbol{\alpha}}^{C} \underset{(7.5)}{\triangleq} \text{\_\_\_\_} \qquad {}^{N}\vec{\boldsymbol{\alpha}}^{C} \underset{(7.1)}{=} \omega_B\,\dot{\theta}\,\hat{\mathbf{b}}_x + \ddot{\theta}\,\hat{\mathbf{b}}_z$$

6.11 ♣ Theorems: Rotation matrices R, angular velocity $\vec{\boldsymbol{\omega}}$, angular acceleration $\vec{\boldsymbol{\alpha}}$? (Section 7.4).

Determine whether or not each theorem to the right is valid for general 3D motion of reference frames A, B, C, and D.

Theorem	True or false
${}^{a}R^{d} = {}^{a}R^{b} * {}^{b}R^{c} * {}^{c}R^{d}$	**True/False**
${}^{A}\vec{\boldsymbol{\omega}}^{D} = {}^{A}\vec{\boldsymbol{\omega}}^{B} + {}^{B}\vec{\boldsymbol{\omega}}^{C} + {}^{C}\vec{\boldsymbol{\omega}}^{D}$	**True/False**
${}^{A}\vec{\boldsymbol{\alpha}}^{D} = {}^{A}\vec{\boldsymbol{\alpha}}^{B} + {}^{B}\vec{\boldsymbol{\alpha}}^{C} + {}^{C}\vec{\boldsymbol{\alpha}}^{D}$	**True/False**

6.12 Alternate formula for angular acceleration. (Section 7.3).

Show/prove ${}^{N}\vec{\boldsymbol{\alpha}}^{B} \triangleq \dfrac{{}^{N}d\;{}^{N}\vec{\boldsymbol{\omega}}^{B}}{dt}$ can also be calculated as ${}^{N}\vec{\boldsymbol{\alpha}}^{B} = \dfrac{{}^{B}d\;{}^{N}\vec{\boldsymbol{\omega}}^{B}}{dt}$.

6.13 Optional: Angular acceleration addition theorem. (Sections 7.3, 7.3.5, 7.4).

Use the angular velocity addition theorem and the definition of angular acceleration to prove:

$$\boxed{{}^{N}\vec{\boldsymbol{\alpha}}^{B} = {}^{N}\vec{\boldsymbol{\alpha}}^{A} + {}^{A}\vec{\boldsymbol{\alpha}}^{B} + {}^{N}\vec{\boldsymbol{\omega}}^{A} \times {}^{A}\vec{\boldsymbol{\omega}}^{B}}$$