

Show work – except for ♣ fill-in-blanks.

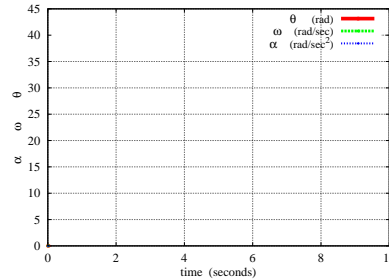
Angular velocity and angular acceleration

6.1 FE/EIT Review – Motion graph: $T \Rightarrow \alpha \Rightarrow \omega \Rightarrow \theta$

A wind turbine generates electricity from wind forces that create a time-dependent torque modeled as $T = 20 \frac{Nm}{sec} * t$. Measures of the wind turbine’s angular acceleration α , angular velocity ω , and angle θ are related as shown below, where $I = 80 \text{ kg m}^2$ is the relevant moment of inertia.

$$T = I \alpha \quad \alpha = \frac{d\omega}{dt} \quad \omega = \frac{d\theta}{dt}$$

Graph α in $\frac{rad}{sec^2}$, ω in $\frac{rad}{sec}$, and θ in rad for $0 \leq t \leq 10 \text{ sec}$. Use initial values (i.e, values at $t = 0$) of $\omega = 0$ and $\theta = 0$.



6.2 Drawing a reference frame and unit vector bases. (Section 8.2)

Draw a rigid body B , shaped like a uniform-density doughnut (having a hole).

Draw a right-handed orthogonal bases fixed in B having unit vectors $\hat{b}_x, \hat{b}_y, \hat{b}_z$.

Draw a different right-handed orthogonal bases fixed in B with unit vectors $\hat{b}_1, \hat{b}_2, \hat{b}_3$.

Draw a properly located center of mass symbol \ominus and label this point as B_{cm} .

Draw a point B_o fixed on B (called B 's origin) at a location different than B_{cm} .

6.3 ♣ Words and pictures for ${}^bR^a$, ${}^{N\vec{\omega}}B$, ${}^{N\vec{\alpha}}B$. (Chapters 5 and 8)

${}^bR^a$ – Description (words)	${}^{N\vec{\omega}}B$ – Description (words)	${}^{N\vec{\alpha}}B$ – Description (words)
[Yellow boxes for description]	[Yellow boxes for description]	[Yellow boxes for description]
Draw b and a	Draw B and N	
[Dashed box for drawing]	[Dashed box for drawing]	

6.4 ♣ Definition of angular velocity? (Section 8.3.3).

The definition of angular velocity of $\vec{\omega} \triangleq \dot{\theta} \vec{k}$ is generally useful for calculating angular velocity and proving its properties (for both 2D and/or 3D analysis). **True/False**

6.5 ♣ Textbook/Internet definitions of 3D angular velocity (Section 8.3).

Famed dynamicist Thomas Kane called angular velocity “**one of the most misunderstood concepts in kinematics.**” Report two definitions of angular velocity and determine if the quantities appearing in the definition are **rigorously defined** - and whether they are generally applicable for 3D kinematics or only apply for simple angular velocity (described in Section 8.3.3).

Note: A definition should be able to **prove** important theorems [such as the angular velocity addition theorem of equation (8.4) and the golden rule for vector differentiation in equation (8.1)] and allow for angular velocity **calculations**.

Source (reference)	Definition	Rigorously defined	Works for 3D kinematics?
List textbook or .html link	Record equation/definition	Yes/No	Yes/No
List textbook or .html link	Record equation/definition	Yes/No	Yes/No

6.6 **Angular velocity of a Ferris-wheel seat** (courtesy of David Levinson).

Given: The rigid seat on a Ferris wheel does not change its orientation relative to ground as the Ferris wheel rotates.

Decide: The seat's angular velocity $\vec{\omega}$ relative to ground (circle one):

Is zero $\vec{\omega} = \vec{0}$	Is constant $\vec{\omega} = \text{Constant} \neq \vec{0}$	Varies $\vec{\omega} = \vec{\omega}(t)$	Does not exist
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6.7 ♣ **Concept: What objects have a unique angular velocity/acceleration?** (Sections 8.3, 8.4).

The angular velocity $\vec{\omega}$ of some object S relative to Earth is to be determined.

This object S could be a (circle **all** objects that have an **unambiguously** defined angular velocity $\vec{\omega}$):

Real number	Matrix	Set of points	Mass center of a rigid body
Vector	Point	Reference frame	Flexible body
3D orthogonal unit basis	Particle	3D rigid body	System of particles and bodies

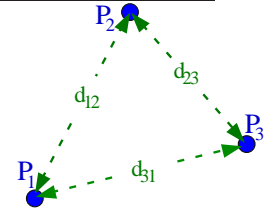
Repeat for the angular acceleration $\vec{\alpha}$ of some object S relative to Earth box appropriate objects.

6.8 ♣ **What is a reference frame, rigid body, and orthogonal basis?** (Sections 4.1 and 8.2)

#	Statement (regard " rigid body " as a massive 2D or 3D rigid object)	True or False
a	A reference frame has all the attributes of a rigid body.	True/False
b	A rigid body has all the attributes of a reference frame.	True/False
c	A reference frame with time-invariant distributed mass is a rigid body.	True/False
d	A massless 3D rigid object is a reference frame.	True/False
e	The definition of a reference frame implies a sense of time.	True/False
f	A rigid body B has a uniquely-defined angular velocity in a reference frame N .	True/False
g	A point Q has a uniquely-defined angular velocity in a reference frame N .	True/False
h	The reference frame B implies unique orthogonal unit vectors $\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$.	True/False
i	The right-handed orthogonal unit vectors $\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$ imply a unique reference frame.	True/False
j	The reference frame B implies a unique rigid frame.	True/False
k	A rigid frame with origin B_o and basis $\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$ implies a unique reference frame.	True/False

6.9 ♣ **Concept: Reference frames and vector bases.** (Sections 4.1 and 8.2)

Consider 3 distinct non-collinear points P_1, P_2, P_3 and the non-zero distances d_{12}, d_{23}, d_{31} between them. In general, determine if each object below can **always** be constructed from P_1, P_2, P_3 under the listed condition.



For each "Yes" answer, **draw** the object.

Condition	Object to be constructed	Object can be constructed?	If Yes, Draw
d_{12}, d_{23}, d_{31} are constant	Vector basis that spans 3D space	Yes/No	
d_{12}, d_{23}, d_{31} are variable	Vector basis that spans 3D space	Yes/No	
d_{12}, d_{23}, d_{31} are constant	Right-handed, orthogonal, unitary basis	Yes/No	
d_{12}, d_{23}, d_{31} are variable	Right-handed, orthogonal, unitary basis	Yes/No	
d_{12}, d_{23}, d_{31} are constant	Unique reference frame	Yes/No	
d_{12}, d_{23}, d_{31} are variable	Unique reference frame	Yes/No	

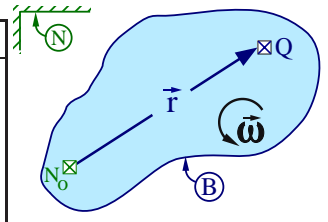
6.10 ♣ **Concepts: What objects have a uniquely-defined angular velocity?** (Section 8.3).

a.	It is possible to find the angular velocity of a point	in a reference frame.	True/False
b.	It is possible to find the angular velocity of a 3D rigid body	in a particle.	True/False
c.	It is possible to find the angular velocity of a 3D rigid body	in a reference frame.	True/False
d.	It is possible to find the angular velocity of a reference frame	in a 3D rigid body.	True/False
d.	It is possible to find the angular velocity of a 3D rigid basis	in a reference frame.	True/False
e.	It is possible to find the angular velocity of a reference frame	in a flexible body.	True/False
f.	It is possible to find the angular velocity of a flexible body	in a reference frame.	True/False

6.11 ♣ **Vector differentiation concepts** “ $v = \omega r$ ”. (Section 8.3).

Point Q is fixed on a rigid body B . Point N_o is fixed in a reference frame N and does not move on B . Complete the following proof that shows how \vec{v} (Q 's velocity in N) can be written in terms of ${}^N\vec{\omega}^B$ (B 's angular velocity in N) and \vec{r} (Q 's position vector from N_o).

Mathematical statement	Reasoning (explain each step below with a brief phrase)
$\vec{v} \triangleq \frac{{}^N d\vec{r}}{dt}$	Definition of Q 's velocity in N
$=$ [] $+$ []	[]
$= \vec{0} + {}^N\vec{\omega}^B \times \vec{r}$	[]



6.12 ♣ **Rotational kinematics of a fire ladder.** (Sections 8.3.3, 8.3.5, 8.3.6).

The following figure shows a fire truck chassis A traveling at constant speed in straight-line motion on Earth (A does not rotate relative to Earth). Earth is a *Newtonian reference frame* N .

A rigid hub B is connected to fire truck A by a revolute motor at point B_o of B .

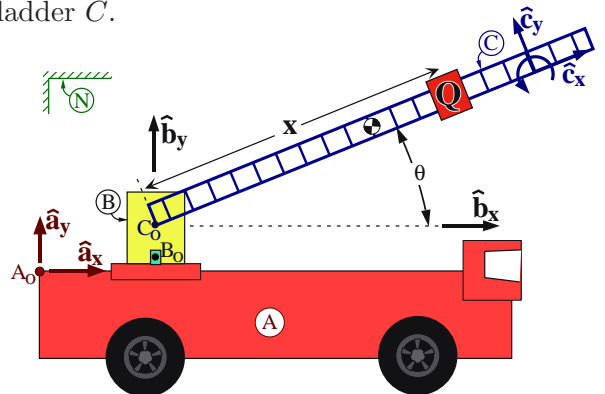
A rigid ladder C is connected to hub B by a revolute motor at point C_o of C .

A fire-fighter Q (modeled as a particle of mass m) climbs ladder C .

Right-handed orthogonal unit vectors $\hat{a}_x, \hat{a}_y, \hat{a}_z$;

$\hat{b}_x, \hat{b}_y, \hat{b}_z$; $\hat{c}_x, \hat{c}_y, \hat{c}_z$; are fixed in A, B, C , with:

- \hat{a}_x pointing forward on the fire truck
- \hat{a}_y vertically-upward and from B_o to C_o
- $\hat{b}_y = \hat{a}_y$ parallel to the axis of the revolute motor that connects B and A
- $\hat{b}_z = \hat{c}_z$ parallel to the axis of the revolute motor that connects B and C
- \hat{c}_x directed from C_o to Q (along C 's long axis)



Note: **Visualize** C 's “**Body yz**” (or “**Space zy**”) rotation sequence in N (e.g., with a ruler).

Quantity	Symbol	Type
\hat{b}_y measure of B 's angular velocity in A	ω_B	Constant
Angle from \hat{b}_x to \hat{c}_x with $+\hat{c}_z$ sense	θ	Variable

${}^cR^b$	[]	[]	[]
[]	[]	[]	[]
[]	[]	[]	[]
[]	[]	[]	[]

- (a) Complete the previous ${}^cR^b$ rotation table (to the right).
Note: ${}^cR^b$ is unnecessary for the remainder of this problem.
- (b) Clarify the process to determine C 's angular velocity in B . (Section 8.3.3).
- ${}^B\vec{\omega}^C$ is a **simple** angular velocity because \hat{b}_z is a vector fixed in **both** [] and [].
 - ${}^B\vec{\omega}^C =$ [], where [] is the angle between [] and [].
 - The sign (\pm) is determined using the []-hand rule (sweep from [] to []).
- (c) ${}^A\vec{\omega}^B = \omega_B \hat{b}_y$ is a **simple** angular velocity because \hat{b}_y is a vector fixed in **both** [] and [].
- (d) Form C 's angular velocity in N and express it in terms of $\hat{b}_x, \hat{b}_y, \hat{b}_z$.

Result:
$${}^N\vec{\omega}^C \stackrel{(8.4)}{=} []\vec{\omega} [] + []\vec{\omega} [] + []\vec{\omega} [] = \vec{0} + []\hat{b}_y + []\hat{b}_z$$

(e) When both ω_B and $\dot{\theta}$ are **constant**, ${}^N\vec{\alpha}^C = \vec{0}$. **True/False**.

(f) Write the definition for C 's angular acceleration in N and form ${}^N\vec{\alpha}^C$. (Sections 8.4, 8.3).

Result:
$${}^N\vec{\alpha}^C \stackrel{(8.8)}{\triangleq} [] \quad {}^N\vec{\alpha}^C \stackrel{(8.1)}{=} \omega_B \dot{\theta} \hat{b}_x + \ddot{\theta} \hat{b}_z$$