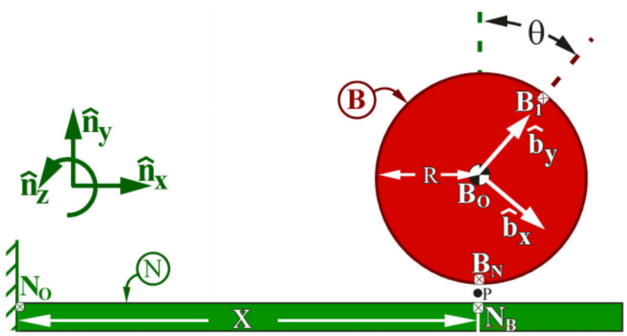
Show work – except for ♣ fill-in-blanks. **Constraints I** (contact, rolling, closed-chains/linkages, ...)

10.1 Velocity and acceleration of a wheel sliding on a plane (Section 11.12).

The following shows a wheel B of radius R in contact with a horizontal road N.
Point N_o is fixed on N.
Point B_o is the wheel's geometric center.
Point B_1 is fixed to B at the wheel's periphery.

Right-handed orthogonal unit vectors $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ and $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$ are fixed in N and B, with

- $\widehat{\mathbf{n}}_x$ horizontally-right
- $\widehat{\mathbf{n}}_y$ vertically-upward
- $\widehat{\mathbf{n}}_z = \widehat{\mathbf{b}}_z$ parallel to B's angular velocity in N
- $\widehat{\mathbf{b}}_y$ directed from B_o to B_1



B's translation in N is characterized by x, the $\widehat{\mathbf{n}}_x$ measure of B_o's position from N_o.
B's rotation in N is characterized by θ, the angle from $\widehat{\mathbf{n}}_y$ to $\widehat{\mathbf{b}}_y$ with $-\widehat{\mathbf{n}}_z$ sense.

Succinctly answer each question in terms of $R, x, \dot{x}, \ddot{x}, \theta, \dot{\theta}, \ddot{\theta}$, $\widehat{\mathbf{n}}_x, \widehat{\mathbf{n}}_y, \widehat{\mathbf{n}}_z$, $\widehat{\mathbf{b}}_x, \widehat{\mathbf{b}}_y, \widehat{\mathbf{b}}_z$.

(a) Determine B's angular velocity in N and B's angular acceleration in N.
Use **definitions** to calculate B_o's velocity in N and B_1's velocity in N.
Result:

$$^N\vec{\boldsymbol{\omega}}^B = \boxed{} \qquad ^N\vec{\boldsymbol{\alpha}}^B = \boxed{} \qquad ^N\vec{\mathbf{v}}^{B_o} = \boxed{} \qquad ^N\vec{\mathbf{v}}^{B_1} = \boxed{}\,\widehat{\mathbf{n}}_x + \boxed{}\,\widehat{\mathbf{b}}_x$$

(b) Find B_1's velocity and acceleration in N at the **instant** when B_1 is in contact with N.
Result:

$$^N\vec{\mathbf{v}}^{B_1}\Big|_{\text{contact}} = (\boxed{} - \boxed{})\,\widehat{\mathbf{n}}_x \qquad ^N\vec{\mathbf{a}}^{B_1}\Big|_{\text{contact}} = (\boxed{} - \boxed{})\,\widehat{\mathbf{n}}_x + \boxed{}\,\widehat{\mathbf{n}}_y$$

(c) Point B_N designates the point **of** B that is in contact with N at each $\boxed{\textbf{instant}}$.
Provide a formula relating $^N\vec{\mathbf{v}}^{B_N}$ to $^N\vec{\mathbf{v}}^{B_o}$ [this formula should not contain a derivative and uses the fact that B_o and B_N are both points **of** (**fixed on**) B]. Then, express $^N\vec{\mathbf{v}}^{B_N}$ in terms of R, $\dot{x}$, $\dot{\theta}$.
Similarly, provide a derivative-free formula relating $^N\vec{\mathbf{a}}^{B_N}$ to $^N\vec{\mathbf{a}}^{B_o}$ and then calculate $^N\vec{\mathbf{a}}^{B_N}$.
Result: (this is a quicker method for calculating the previous results)

$$^N\vec{\mathbf{v}}^{B_N} \underset{(10.3)}{=} {^N\vec{\mathbf{v}}^{B_o}} + \boxed{} \times \boxed{} = (\boxed{} - \boxed{})\,\widehat{\mathbf{n}}_x$$

$$^N\vec{\mathbf{a}}^{B_N} \underset{(10.4)}{=} {^N\vec{\mathbf{a}}^{B_o}} + \boxed{} \times \boxed{} + \boxed{} = (\ddot{x} - R\,\ddot{\theta})\,\widehat{\mathbf{n}}_x + \dot{\theta}^2\,R\,\widehat{\mathbf{n}}_y$$

(d) Point B_N's position vector from N_o is ***always*** $\vec{\mathbf{r}}^{\,B_N/N_o} = x\,\widehat{\mathbf{n}}_x$.
Why is your previous result for $^N\vec{\mathbf{v}}^{B_N}$ **not** the time-derivative in N of $\vec{\mathbf{r}}^{\,B_N/N_o}$?
Why is your previous result for $^N\vec{\mathbf{a}}^{B_N}$ **not** the time-derivative in N of $^N\vec{\mathbf{v}}^{B_N}$?

$$^N\vec{\mathbf{v}}^{B_N} = (\dot{x} - R\,\dot{\theta})\,\widehat{\mathbf{n}}_x \neq \frac{^N d\,\vec{\mathbf{r}}^{\,B_N/N_o}}{dt}$$

$$^N\vec{\mathbf{a}}^{B_N} = (\ddot{x} - R\,\ddot{\theta})\,\widehat{\mathbf{n}}_x + \dot{\theta}^2\,R\,\widehat{\mathbf{n}}_y \neq \frac{^N d\;^N\vec{\mathbf{v}}^{B_N}}{dt}$$

Explain:

(e)

- The symbols B_N and B_1 sometimes designate the same point. **True/False**
- Point B_1 is always the name of the same (one) specific point of B. **True/False**
- Point B_N is always the name of the same (one) specific point of B. **True/False**
- Point B_N is ***continuously renamed*** as B rotates in N. **True/False**
- When B_1 is in contact with N, ${}^{N}\vec{\mathbf{v}}^{B_1}\big|_{\text{contact}}$ has the same meaning as ${}^{N}\vec{\mathbf{v}}^{B_N}$. **True/False**

(f) Calculate N_o's velocity and acceleration in B.
Result:

$${}^{B}\vec{\mathbf{v}}^{N_o} = (\boxed{} - \boxed{})\,\widehat{\mathbf{n}}_x - \boxed{}\,\widehat{\mathbf{n}}_y$$
$${}^{B}\vec{\mathbf{a}}^{N_o} = (x\,\dot\theta^2 + R\,\ddot\theta - \ddot x)\,\widehat{\mathbf{n}}_x + (R\,\dot\theta^2 - 2\,\dot\theta\,\dot x - x\,\ddot\theta)\,\widehat{\mathbf{n}}_y$$

(g) Point N_B designates the point **of** N that is in contact with B at each instant.
Find N_B's velocity and acceleration in B.
Result: ${}^{B}\vec{\mathbf{v}}^{N_B} = (\boxed{} - \boxed{})\,\widehat{\mathbf{n}}_x$ ${}^{B}\vec{\mathbf{a}}^{N_B} = (R\,\ddot\theta - \ddot x)\,\widehat{\mathbf{n}}_x + (R\,\dot\theta^2 - 2\,\dot\theta\,\dot x)\,\widehat{\mathbf{n}}_y$

(h) Define "path point" P as the point that continuously traces out contact between B and N.[1]
Find P's velocity and acceleration in N and P's velocity and acceleration in B.
Result: ${}^{N}\vec{\mathbf{v}}^{P} = \boxed{}$ ${}^{N}\vec{\mathbf{a}}^{P} = \ddot x\,\widehat{\mathbf{n}}_x$

${}^{B}\vec{\mathbf{v}}^{P} = \boxed{}$ ${}^{B}\vec{\mathbf{a}}^{P} = R\,\ddot\theta\,\widehat{\mathbf{n}}_x + R\,\dot\theta^2\,\widehat{\mathbf{n}}_y$

(i)

- Points B_N and N_B **never/sometimes/always** designate the same point.
- Points B_N and N_B are **never/sometimes/always** coincident (co-located).
- Points B_N and P **never/sometimes/always** designate the same point.
- Points B_N and P are **never/sometimes/always** coincident (co-located).

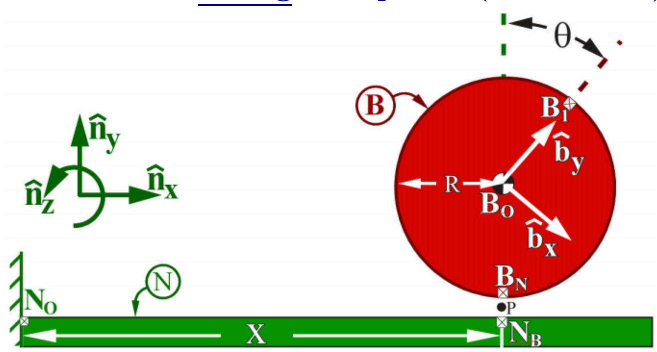
10.2 FE/EIT Review – Velocity and acceleration of a wheel rolling on a plane (Section 11.12).

A thin wheel B of radius R remains in friction contact with a flat horizontal road N.

B has a simple angular velocity parallel to $\widehat{\mathbf{n}}_z$ ($\widehat{\mathbf{n}}_z$ is perpendicular to the circular portion of B).

The point **of** B in contact with N at each ***instant*** is denoted B_N. The point **of** N in contact with B at each ***instant*** is N_B.

Answer questions below with symbols from Homework 10.1, (e.g., R, θ, $\dot\theta$, $\ddot\theta$, $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$).



(a) Write the vector definition of ***rolling*** between N and B. Use it to relate $\dot x$ to R, $\dot\theta$.
Result: $\boxed{} \underset{\text{(rolling)}}{\triangleq} \boxed{}$ $\underset{\text{show work}}{\Rightarrow}$ $\dot x = \boxed{}$

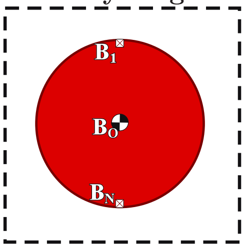
(b) If B **continuously** rolls on N, can $\dot x = R\,\dot\theta$ be differentiated to calculate $\ddot x = R\,\ddot\theta$. **Yes/No**.
For continuous rolling, solve for $x(t)$ in terms of $\theta(t)$ and the initial value $x(0)$ (value of x at $t = 0$).
Result: [use $\theta(0) = 0$] $x(t) = x(0) + \boxed{}$

(c) Calculate the following. Herein, regard B_1 as the point of B at the top of the wheel ($\theta = 0$).
Next, **draw** the velocities and accelerations on the wheel when $\dot\theta$ is constant ($\ddot\theta = 0$).
Result: (Express results **solely** in terms of R, $\dot\theta$, $\ddot\theta$, $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ - **without** x, $\dot x$, $\ddot x$.)

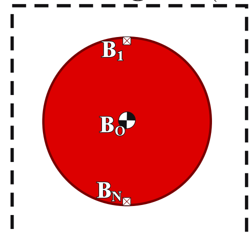
[1] Points **of** B depend on the existence of B. Points **of** N depend on the existence of N. As defined, the existence of "path point" P depends on **contact** between B and N. As soon as contact ceases, point P ceases to exist.

Velocity	${}^N\vec{\mathbf{v}}^{B_N} = \boxed{}$	${}^N\vec{\mathbf{v}}^{B_o} = \boxed{}$	${}^N\vec{\mathbf{v}}^{B_1} = \boxed{}\,\widehat{\mathbf{n}}_x$
Acceleration	${}^N\vec{\mathbf{a}}^{B_N} = R\,\dot{\theta}^2\,\widehat{\mathbf{n}}_y$	${}^N\vec{\mathbf{a}}^{B_o} = \boxed{}$	${}^N\vec{\mathbf{a}}^{B_1} = \boxed{}\,\widehat{\mathbf{n}}_x + \boxed{}\,\widehat{\mathbf{n}}_y$

Velocity diagram



Acceleration diagram (when $\ddot{\theta} = 0$)



Answer the following. Assume continuous rolling or sliding. Assume non-zero motion of B in N (either $\dot{\theta} \neq 0$ or $\dot{x} \neq 0$).

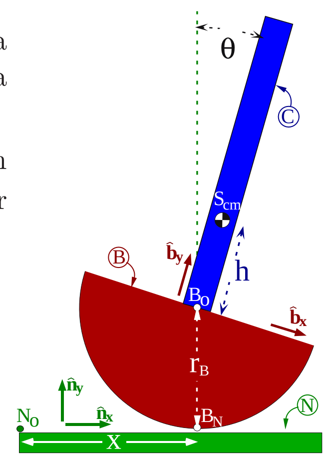
- When B ***rolls*** on N, the velocity of B_N in N ***must be*** zero. **True/False**
- When B ***slides*** on N, the velocity of B_N in N ***must be*** zero. **True/False**
- When B ***rolls*** on N, the acceleration of B_N in N ***can be*** zero. **True/False**
- When B ***slides*** on N, the acceleration of B_N in N ***can be*** zero. **True/False**

† Prove your previous answer: If ${}^N\vec{\mathbf{a}}^{B_N} = \vec{\mathbf{0}}$, can B ***continuously slide*** on N? Determine $\dot{\theta}$, $\ddot{\theta}$, $\ddot{x}$, $\dot{x}$.
Result: $\dot{\theta}(t) = \boxed{}$ $\quad\ddot{\theta}(t) = \boxed{}$ $\quad\ddot{x}(t) = \boxed{}$ $\quad\dot{x}(t) = \boxed{}$ Slides? **Yes/No**

10.3 Kinematics of a rocking/rolling sailboat (Section 11.12).

The figure to the right shows a uniform rigid rod C that is welded to a uniform rigid half-cylinder B at point B_o. Body B is in contact with a horizontal plane N at point B_N of B. Point N_o is fixed in N.

Right-handed orthogonal unit vectors $\widehat{\mathbf{b}}_x$, $\widehat{\mathbf{b}}_y$, $\widehat{\mathbf{b}}_z$ and $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ are fixed in B and N, respectively, with $\widehat{\mathbf{n}}_y$ vertically-upward, $\widehat{\mathbf{n}}_z = \widehat{\mathbf{b}}_z$ perpendicular to B's flat semi-circular portion, and $\widehat{\mathbf{b}}_y$ parallel to the rod's long axis.



Quantity	Symbol	Type
Radius of B	r_B	Constant
Distance between B_o and S_{cm}	h	Constant
Angle from $\widehat{\mathbf{b}}_y$ to $\widehat{\mathbf{n}}_y$ with $^+\widehat{\mathbf{n}}_z$ sense	θ	Variable
$\widehat{\mathbf{n}}_x$ measure of B_N's position vector from N_o	x	Variable

Results from Homework 8.11 include: ${}^N\vec{\boldsymbol{\omega}}^B = -\dot{\theta}\,\widehat{\mathbf{b}}_z$ and ${}^N\vec{\mathbf{v}}^{B_o} = \dot{x}\,\widehat{\mathbf{n}}_x$.

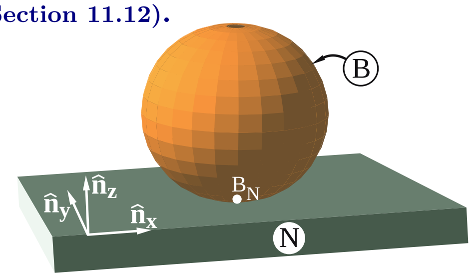
Now assume B ***rolls*** on N at point B_N of B (there is sufficient friction between N and B for rolling).
Show ***every*** step that relates $\dot{x}$ to $\dot{\theta}$, r_B. Then express $\ddot{x}$ in terms of $\ddot{\theta}$.
Next, determine B_N's acceleration in N and express it without x, $\dot{x}$, or $\ddot{x}$.
Result: $\dot{x} = \boxed{}$ $\quad\ddot{x} = \boxed{}$ $\quad{}^N\vec{\mathbf{a}}^{B_N} = \dot{\theta}^2\,r_B\,\widehat{\mathbf{n}}_y$

When B ***rolls*** on N: ${}^N\vec{\mathbf{v}}^{B_N} = \vec{\mathbf{0}}$ **True/False**; ${}^N\vec{\mathbf{a}}^{B_N} = \vec{\mathbf{0}}$ **True/False**.

Note: This system's equation of motion is formed in Homework 20.12 and Homework 22.1.

10.4 Velocity and acceleration of a sliding/rolling sphere (Section 11.12).

The figure to the right shows a sphere B of radius R moving on a horizontal plane N. B's angular velocity in N and the velocity of B_o (B's geometric center) in N are expressed in terms of time-dependent variables ω_x, ω_y, ω_z, v_x, v_y as



$$
\begin{aligned}
{}^N\vec{\boldsymbol{\omega}}^B &= \omega_x\,\widehat{\mathbf{n}}_x + \omega_y\,\widehat{\mathbf{n}}_y + \omega_z\,\widehat{\mathbf{n}}_z \\
{}^N\vec{\mathbf{v}}^{B_o} &= v_x\,\widehat{\mathbf{n}}_x + v_y\,\widehat{\mathbf{n}}_y
\end{aligned}
$$