

Show work – except for ♣ fill-in-blanks.

Constraints I (contact, rolling, closed-chains/linkages, ...)

10.1 Velocity and acceleration of a wheel sliding on a plane (Section 11.12).

The following shows a wheel B of radius R in contact with a horizontal road N .

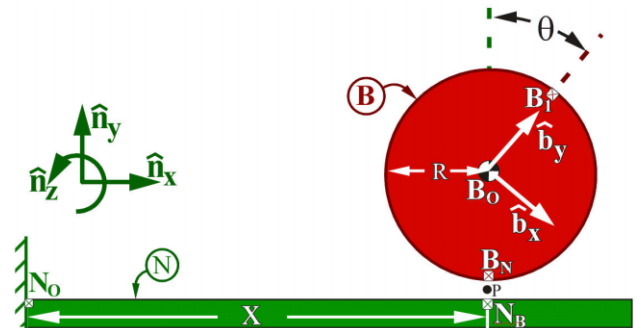
Point N_o is fixed on N .

Point B_o is the wheel's geometric center.

Point B_1 is fixed to B at the wheel's periphery.

Right-handed orthogonal unit vectors $\hat{n}_x, \hat{n}_y, \hat{n}_z$ and $\hat{b}_x, \hat{b}_y, \hat{b}_z$ are fixed in N and B , with

- \hat{n}_x horizontally-right
- \hat{n}_y vertically-upward
- $\hat{n}_z = \hat{b}_z$ parallel to B 's angular velocity in N
- \hat{b}_y directed from B_o to B_1



B 's translation in N is characterized by x , the \hat{n}_x measure of B_o 's position from N_o .

B 's rotation in N is characterized by θ , the angle from \hat{n}_y to \hat{b}_y with $-\hat{n}_z$ sense.

Succinctly answer each question in terms of $R, x, \dot{x}, \ddot{x}, \theta, \dot{\theta}, \ddot{\theta}, \hat{n}_x, \hat{n}_y, \hat{n}_z, \hat{b}_x, \hat{b}_y, \hat{b}_z$.

- (a) Determine B 's angular velocity in N and B 's angular acceleration in N .
Use **definitions** to calculate B_o 's velocity in N and B_1 's velocity in N .

Result:

$${}^N\vec{\omega}^B = \text{[]} \quad {}^N\vec{\alpha}^B = \text{[]} \quad {}^N\vec{v}^{B_o} = \text{[]} \quad {}^N\vec{v}^{B_1} = \text{[]}$$

- (b) Find B_1 's velocity in N at the **instant** when B_1 is in contact with N .

Result: (in terms of $\hat{n}_x, \hat{n}_y, \hat{n}_z$)

$${}^N\vec{v}^{B_1} \Big|_{\text{contact}} = \text{[]}$$

- (c) Point B_N designates the point **of** B that is in contact with N at each **instant**.

Provide a formula relating ${}^N\vec{v}^{B_N}$ to ${}^N\vec{v}^{B_o}$ [this formula should not contain a derivative and uses the fact that B_o and B_N are both points **of** (fixed on) B]. Then, express ${}^N\vec{v}^{B_N}$ in terms of $R, \dot{x}, \dot{\theta}$.

Similarly, provide a derivative-free formula relating ${}^N\vec{a}^{B_N}$ to ${}^N\vec{a}^{B_o}$ and then calculate ${}^N\vec{a}^{B_N}$.

Result:

$${}^N\vec{v}^{B_N} = {}^N\vec{v}^{B_o} + \text{[]} \times \text{[]} = (\text{[]} - \text{[]}) \hat{n}_x \tag{10.3}$$

$${}^N\vec{a}^{B_N} = {}^N\vec{a}^{B_o} + \text{[]} \times \text{[]} + \text{[]} = (\ddot{x} - R\ddot{\theta}) \hat{n}_x + \dot{\theta}^2 R \hat{n}_y \tag{10.4}$$

- (d) Point B_N 's position vector from N_o is **always** $\vec{r}^{B_N/N_o} = x \hat{n}_x$.

Why is your previous result for ${}^N\vec{v}^{B_N}$ **not** the time-derivative in N of \vec{r}^{B_N/N_o} ?

Why is your previous result for ${}^N\vec{a}^{B_N}$ **not** the time-derivative in N of ${}^N\vec{v}^{B_N}$?

$${}^N\vec{v}^{B_N} = (\dot{x} - R\dot{\theta}) \hat{n}_x \neq \frac{{}^N d \vec{r}^{B_N/N_o}}{dt}$$

$${}^N\vec{a}^{B_N} = (\ddot{x} - R\ddot{\theta}) \hat{n}_x + \dot{\theta}^2 R \hat{n}_y \neq \frac{{}^N d {}^N\vec{v}^{B_N}}{dt}$$

Explain:

[]

[]

[]

- The symbols B_N and B_1 sometimes designate the same point. True/False
 - Point B_1 is always the name of the same (one) specific point of B . True/False
- (e)
- Point B_N is always the name of the same (one) specific point of B . True/False
 - Point B_N is continuously renamed as B rotates in N . True/False
 - When B_1 is in contact with N , ${}^{N}\vec{v}^{B_1}|_{\text{contact}}$ has the same meaning as ${}^{N}\vec{v}^{B_N}$. True/False

(f) Calculate N_o 's velocity in B and N_o 's acceleration in B .

Result:

$${}^B\vec{v}^{N_o} = \text{[]} \hat{n}_x - \text{[]} \hat{n}_y$$

$${}^B\vec{a}^{N_o} = (x\dot{\theta}^2 + R\ddot{\theta} - \ddot{x})\hat{n}_x + (R\dot{\theta}^2 - 2\dot{\theta}\dot{x} - x\ddot{\theta})\hat{n}_y$$

(g) Point N_B designates the point **of** N that is in contact with B at each instant. Find N_B 's velocity in B and N_B 's acceleration in B .

Result:

$${}^B\vec{v}^{N_B} = \text{[]} \quad {}^B\vec{a}^{N_B} = (R\ddot{\theta} - \ddot{x})\hat{n}_x + (R\dot{\theta}^2 - 2\dot{\theta}\dot{x})\hat{n}_y$$

(h) Point P designates the point that continuously traces out contact between B and N .¹ Find P 's velocity and acceleration in N and P 's velocity and acceleration in B .

Result:

$${}^N\vec{v}^P = \text{[]} \quad {}^N\vec{a}^P = \ddot{x}\hat{n}_x$$

$${}^B\vec{v}^P = \text{[]} \quad {}^B\vec{a}^P = R\ddot{\theta}\hat{n}_x + R\dot{\theta}^2\hat{n}_y$$

- | | | | |
|-----|------------------------------|------------------------|---------------------------|
| | • Symbols B_N and N_B | never/sometimes/always | designate the same point. |
| (i) | • Points B_N and N_B are | never/sometimes/always | coincident. |
| | • Symbols B_N and P | never/sometimes/always | designate the same point. |
| | • Points B_N and P are | never/sometimes/always | coincident. |

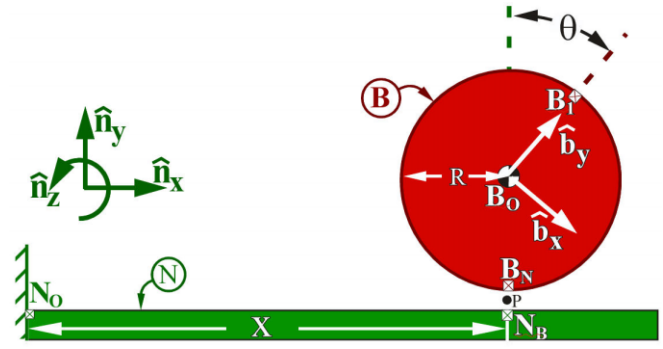
10.2 FE/EIT Review – Velocity and acceleration of a wheel rolling on a plane (Section 11.12).

A thin wheel B of radius R remains in contact with a flat horizontal road N .

B has a simple angular velocity parallel to \hat{n}_z (\hat{n}_z is perpendicular to the circular portion of B).

The point **of** B in contact with N at each **instant** is denoted B_N . The point **of** N in contact with B at each **instant** is N_B .

Answer questions below with symbols from Homework 10.1, (e.g., $R, \theta, \dot{\theta}, \ddot{\theta}, \hat{n}_x, \hat{n}_y, \hat{n}_z$).



(a) Write the vector definition of **rolling** between N and B . Use it to relate \dot{x} to R and $\dot{\theta}$.

Result:

$$\text{[]} \stackrel{\triangle}{=} \text{[]} \quad \Rightarrow \quad \dot{x} = \text{[]}$$

(rolling)

(b) If B **continuously** rolls on N , can $\dot{x} = R\dot{\theta}$ be differentiated to calculate $\ddot{x} = R\ddot{\theta}$. **Yes/No.** For continuous rolling, solve for $x(t)$ in terms of $\theta(t)$ and the initial value $x(0)$ (value of x at $t = 0$).

Result:

$$x(t) = x(0) + \text{[]}$$

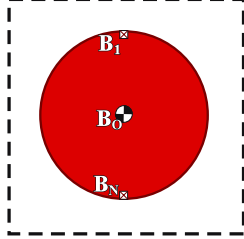
(c) Calculate the following. Herein, regard B_1 as the point of B at the top of the wheel ($\theta = 0$). Next, **draw** the velocities and accelerations on the wheel when $\dot{\theta}$ is constant ($\ddot{\theta} = 0$).

Result: (Express results **solely** in terms of $R, \dot{\theta}, \ddot{\theta}, \hat{n}_x, \hat{n}_y, \hat{n}_z$ - **without** x, \dot{x}, \ddot{x} .)

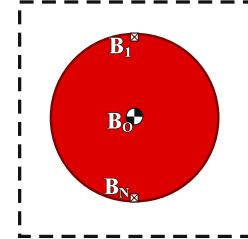
¹Although points **of** B depend on the existence of B and points **of** N depend on the existence of N , the existence of “path point” P depends on **contact** between B and N . As soon as contact ceases, point P ceases to exist.

Velocity	$N\vec{v}^{B_N} =$ <input type="text"/>	$N\vec{v}^{B_o} =$ <input type="text"/>	$N\vec{v}^{B_1} =$ <input type="text"/>
Acceleration	$N\vec{a}^{B_N} = R\dot{\theta}^2 \hat{n}_y$	$N\vec{a}^{B_o} =$ <input type="text"/>	$N\vec{a}^{B_1} =$ <input type="text"/> \hat{n}_x + <input type="text"/> \hat{n}_y

Velocity diagram



Acceleration diagram (when $\ddot{\theta} = 0$)



Answer the following. Assume rolling and sliding imply non-zero motion of B in N (i.e., non-zero $\dot{\theta}$ and/or \dot{x}).

- When B **rolls** on N , the velocity of B_N in N **must be** zero. **True/False**
- When B **slides** on N , the velocity of B_N in N **must be** zero. **True/False**
- When B **rolls** on N , the acceleration of B_N in N **can be** zero. **True/False**
- When B **slides** on N , the acceleration of B_N in N **can be** zero. **True/False**

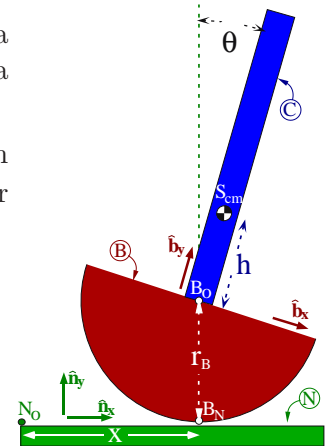
If B continuously **slides** on N and $N\vec{a}^{B_N} = \vec{0}$, determine the values/conditions on \dot{x} , \ddot{x} , $\dot{\theta}$, $\ddot{\theta}$.

Result: $\dot{x}(t) =$ $\ddot{x}(t) =$ $\dot{\theta}(t) =$ $\ddot{\theta}(t) =$

10.3 Kinematics of a rocking sailboat (Section 11.12).

The figure to the right shows a uniform rigid rod C that is welded to a uniform rigid half-cylinder B at point B_o . Body B is in contact with a **rough** horizontal plane N at point B_N of B . Point N_o is fixed in N .

Right-handed orthogonal unit vectors $\hat{b}_x, \hat{b}_y, \hat{b}_z$ and $\hat{n}_x, \hat{n}_y, \hat{n}_z$ are fixed in B and N , respectively, with \hat{n}_y vertically-upward, $\hat{n}_z = \hat{b}_z$ perpendicular to B 's flat semi-circular portion, and \hat{b}_y parallel to the rod's long axis.



Quantity	Symbol	Type
Radius of B	r_B	Constant
Distance between B_o and S_{cm}	h	Constant
Angle from \hat{b}_y to \hat{n}_y with $+\hat{n}_z$ sense	θ	Variable
\hat{n}_x measure of B_N 's position vector from N_o	x	Variable

Results from Homework 8.11 include: $N\vec{\omega}^B = -\dot{\theta}\hat{b}_z$ and $N\vec{v}^{B_o} = \dot{x}\hat{n}_x$.

For this problem, assume B **rolls** on N at point B_N of B .

Show **every** step that relates \dot{x} to $\dot{\theta}$, r_B . Then express \ddot{x} in terms of $\ddot{\theta}$.

Next, determine B_N 's acceleration in N and express it without x , \dot{x} , or \ddot{x} .

Result: $\dot{x} =$ $\ddot{x} =$ $N\vec{a}^{B_N} = \dot{\theta}^2 r_B \hat{n}_y$

When B **rolls** on N : $N\vec{v}^{B_N} = \vec{0}$ **True/False**; $N\vec{a}^{B_N} = \vec{0}$ **True/False**.

Note: This system's equation of motion is formed in Homework 20.15 and Homework 22.1.

10.4 Velocity and acceleration of a sliding/rolling sphere (Section 11.12).

The figure to the right shows a sphere B of radius R moving on a horizontal plane N . The velocity of B_o (B 's geometric center) in N and B 's angular velocity in N are expressed in terms of time-dependent variables $v_x, v_y, \omega_x, \omega_y, \omega_z$ as

$$N\vec{v}^{B_o} = v_x \hat{n}_x + v_y \hat{n}_y$$

$$N\vec{\omega}^B = \omega_x \hat{n}_x + \omega_y \hat{n}_y + \omega_z \hat{n}_z$$

