

11.1 ♣ Velocity variables and degrees of freedom (Section 11.1).

Determine the minimum number of unknown *velocity variables* necessary to characterize the motion of the following systems in a reference frame N . Regard Q as a free-flying particle and A as a rigid body that is **free** to translate and rotate in 3D-space. Choose and **define** velocity variables that suffice to describe the motion (Note: The choice of velocity variables is **not unique**).

Optional: **Sketch** each system with names for each point/body.

System (Q , B , or A and B)	Degrees of freedom	Choice of velocity variables
Free-flying particle Q .	3	v_x v_y v_z ${}^N\vec{v}^Q = v_x \hat{n}_x + v_y \hat{n}_y + v_z \hat{n}_z$
Particle Q moving in a slot (slot is parallel to a unit vector \hat{n}).	1	v ${}^N\vec{v}^Q = v \hat{n}$
Free-flying rigid body B .	6	ω_x ω_y ω_z v_x v_y v_z ${}^N\vec{\omega}^B = \omega_x \hat{b}_x + \omega_y \hat{b}_y + \omega_z \hat{b}_z$ ${}^N\vec{v}^{B_o} = v_x \hat{n}_x + v_y \hat{n}_y + v_z \hat{n}_z$
Rigid body B connected to rigid body A by a revolute joint. (A connects to B at point A_B of A)	7	ω_x ω_y ω_z v_x v_y v_z ω_B ${}^N\vec{\omega}^A = \omega_x \hat{a}_x + \omega_y \hat{a}_y + \omega_z \hat{a}_z$ ${}^N\vec{v}^{A_B} = v_x \hat{a}_x + v_y \hat{a}_y + v_z \hat{a}_z$ ${}^A\vec{\omega}^B = \omega_B \hat{\lambda}$
Rigid body B connected to rigid body A by a rigid joint.	6	ω_x ω_y ω_z v_x v_y v_z ${}^N\vec{\omega}^A = \omega_x \hat{a}_x + \omega_y \hat{a}_y + \omega_z \hat{a}_z$ ${}^N\vec{v}^{A_B} = v_x \hat{n}_x + v_y \hat{n}_y + v_z \hat{n}_z$
Rigid body B connected to A by a ball-and-socket joint.	9	ω_x ω_y ω_z v_x v_y v_z ω_x^B ω_y^B ω_z^B ${}^N\vec{\omega}^A = \omega_x \hat{a}_x + \omega_y \hat{a}_y + \omega_z \hat{a}_z$ ${}^N\vec{v}^{A_B} = v_x \hat{a}_x + v_y \hat{a}_y + v_z \hat{a}_z$ ${}^A\vec{\omega}^B = \omega_x^B \hat{b}_x + \omega_y^B \hat{b}_y + \omega_z^B \hat{b}_z$
Rigid body B connected to A by a revolute angular velocity motor. (A revolute angular velocity motor <i>specifies</i> B 's angular velocity in A)	6	ω_x ω_y ω_z v_x v_y v_z ${}^N\vec{\omega}^A = \omega_x \hat{a}_x + \omega_y \hat{a}_y + \omega_z \hat{a}_z$ ${}^N\vec{v}^{A_B} = v_x \hat{n}_x + v_y \hat{n}_y + v_z \hat{n}_z$
Rectangular box B sliding on a flat rigid surface fixed in N . (B contacts N at a single vertex B_o of B)	5	ω_x ω_y ω_z v_x v_y ${}^N\vec{v}^{B_o} = v_x \hat{n}_x + v_y \hat{n}_y$ ${}^N\vec{\omega}^B = \omega_x \hat{b}_x + \omega_y \hat{b}_y + \omega_z \hat{b}_z$
Rectangular box B sliding on a flat rigid surface fixed in N . (B contacts N on a single edge of B . The edge is parallel to \hat{b}_x) (Flat surface is perpendicular to \hat{n}_z)	4	ω_y ω_z v_x v_y ${}^N\vec{v}^{B_o} = v_x \hat{n}_x + v_y \hat{n}_y$ ${}^N\vec{\omega}^B = \omega_y \hat{b}_y + \omega_z \hat{b}_z$
Rectangular box B sliding on a flat rigid surface fixed in N . (B contacts N on one surface of B)	3	ω_z v_x v_y ${}^N\vec{v}^{B_o} = v_x \hat{n}_x + v_y \hat{n}_y$ ${}^N\vec{\omega}^B = \omega_z \hat{b}_z$