♣ Optional Demo: How product of inertia affects a rotating rattleback (student version).

From earliest history, children have skipped stones and spun tops. A spinning top's scientific challenge is its curious motions. The ***rattleback*** (also called a ***dynamic celt*** or ***wobblestone***) provides visual, tactile, and auditory insights into ***product of inertia*** and how it affects rotational motion. Beyond its educational value, the rattleback is a static/kinetic friction-verification test for professional motion software.[1]

Product of inertia affects spin motion.

Place a rattleback on a flat hard surface so it looks as shown right (**zero** product of inertia). Spin it **clockwise** and observe its motion. Next, spin it **counter-clockwise**. Circle the best description of the resulting motions.

Initial motion	Best description of resulting motion	
Clockwise spin	Simple spin	Spin reversal and wobble
Counter-clockwise spin	Simple spin	Spin reversal and wobble

Product of inertia: **Zero**

Adjust the bar so it has a **negative** product of inertia (so the bar $\approx 20^\circ$ from the rattleback's long-axis). Spin the rattleback **clockwise** and observe its motion. Next, spin it **counter-clockwise**.

Initial motion	Best description of resulting motion	
Clockwise spin	Simple spin	Spin reversal and wobble
Counter-clockwise spin	Simple spin	Spin reversal and wobble

Product of inertia: **Negative**

Adjust the bar so it has a **positive** product of inertia (as shown right). Spin the rattleback **clockwise**. Next, spin it **counter-clockwise**.

Initial motion	Best description of resulting motion	
Clockwise spin	Simple spin	Spin reversal and wobble
Counter-clockwise spin	Simple spin	Spin reversal and wobble

Adjust the bar so it makes a 5° angle with the rattleback's long-axis. Spin it **counter-clockwise**. Readjust the bar-angle to 45° and repeat. The bar-angle with largest amplitude wobble is $5^\circ / 45^\circ$ (circle one).

Product of inertia: **Positive**

Product of inertia affects pitch motion.

Place a rattleback on a flat hard surface so it looks as shown right. Touch its end so it rocks up and down.

Best description of resulting motion		
Mostly pitch	Pitch and clockwise spin	Pitch and counter-clockwise spin

Product of inertia: **Zero**

Now, adjust the bar so that it has a **negative** product of inertia. Touch its end so it rocks up and down.

Best description of resulting motion		
Mostly pitch	Pitch and clockwise spin	Pitch and counter-clockwise spin

Product of inertia: **Negative**

Next, adjust the bar so that it has a **positive** product of inertia. Touch its end so it rocks up and down.

Best description of resulting motion		
Mostly pitch	Pitch and clockwise spin	Pitch and counter-clockwise spin

Product of inertia: **Positive**

[1] For a video demonstration, visit https://www.youtube.com/watch?v=11NHjiEYnI0 (Tim's grand illusions).

Product of inertia affects roll motion.

Place a rattleback on a flat hard surface so it looks as shown right.
Press its side so it **strongly** rolls from side to side.

Best description of resulting motion		
Mostly pitch	Pitch and clockwise spin	Pitch and counter-clockwise spin
Mostly roll	Roll and weak clockwise spin	Roll and weak counter-clockwise spin

Product of inertia: **Zero**

Now, adjust the bar so that it has a **negative** product of inertia.
Press its side so it **strongly** rolls from side to side.

Best description of resulting motion		
Mostly pitch	Pitch and clockwise spin	Pitch and counter-clockwise spin
Mostly roll	Roll and weak clockwise spin	Roll and weak counter-clockwise spin

Product of inertia: **Negative**

Next, adjust the bar so that it has a **positive** product of inertia.
Press its side so it **strongly** rolls from side to side.

Best description of resulting motion		
Mostly pitch	Pitch and clockwise spin	Pitch and counter-clockwise spin
Mostly roll	Roll and weak clockwise spin	Roll and weak counter-clockwise spin

Product of inertia: **Positive**

Summary of spin, pitch, and roll preferences

Consider a rattleback with a **negative** product of inertia. For each initial motion, circle the final **spin** direction.

Initial motion	Preferred spin direction	
Spin	Clockwise	Counter-clockwise
Pitch	Clockwise	Counter-clockwise
Roll	Clockwise	Counter-clockwise

Product of inertia: **Negative**

Consider a rattleback with a **positive** product of inertia. For each initial motion, circle the final **spin** direction.

Initial motion	Preferred spin direction	
Spin	Clockwise	Counter-clockwise
Pitch	Clockwise	Counter-clockwise
Roll	Clockwise	Counter-clockwise

Product of inertia: **Positive**

How it works?

For more than a century, the rattleback has attracted the attention of eminent physicists. Most agree that its curious behavior results from three things:

- Friction at the point of contact between rattleback and surface
- A curved surface with two different radii of curvature
- Principal axes of **curvature** that misalign with the principal axes of **inertia** (i.e., there is a non-zero ***product of inertia***).

Other related concepts are gravity and normal forces, centrifugal and Coriolis accelerations, and energy exchange between spin, pitching oscillations, and rolling oscillations.

The rattleback shown here was designed to have a perfect ellipsoidly-shaped bottom with easily-identifiable principal axes of **curvature**. The principal **inertia** axes are adjusted by the bar (adjusting the bar does not change the mass or mass center location). Note: Many rattlebacks have grossly imperfect non-ellipsoidal geometry.

Although detailed mathematical analyses accurately predict the rattleback's behavior, there is still great difficultly in understanding the mathematics in physical terms. Quoting Professor Mont Hubbard (who published detailed experimental and theoretical work on the rattleback): "**I don't intuitively understand it**".

Perhaps the best answer to "why does the rattleback move like this" is $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$.
Physics, math, and simulation: www.MotionGenesis.com ⇒ Get Started ⇒ Rattleback (dynamic celt).