Homework 14. Chapters 19, 20.

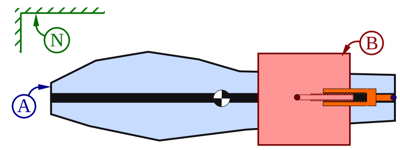
$\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ FBD, MG road-maps, and concepts for translating mechanical systems.

14.1 Concept: Translational motion? (free-body diagrams)

A rigid body B is connected to a rigid body A with a force actuator (pushes B apart from A). Initially, A and B are **at rest** (stationary) in deep empty space in a Newtonian (inertial) reference frame N.

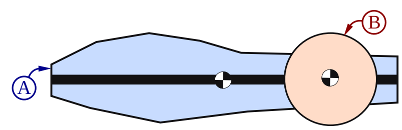
Using physical intuition, guess if the force actuator can move:

• A's mass center in N?	**Yes/No**	
• B's mass center in N?	**Yes/No**	
• System mass center in N?	**Yes/No**	

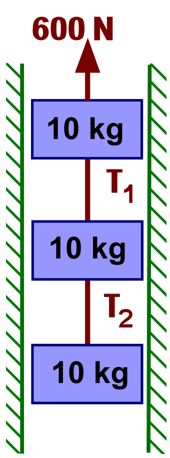


Verify each guess with an equation of motion (3^{rd}-column above).

The previous 3 answers are the **same/different** if B is connected to A with a torque/rotational motor (instead of a force actuator).



14.2 FE/EIT Review – Tension in vertical ropes (free-body diagrams). $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$



$T_1 = T_2$
True/False

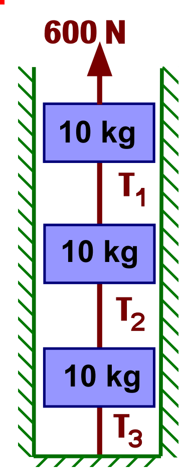
The figure to the left shows three 10 kg rigid blocks that are in a smooth (**frictionless**) vertical slot. The set of interconnected blocks is pulled vertically-upward with a 600 N force.

Under the figure to the left, circle **True** if the tensions T_i (i = 1, 2) in the light (massless) inextensible ropes are equal. Otherwise, circle **False** and report numerical values for T_1 and T_2 below. Approximate Earth's gravitational acceleration as $g = 10\ \frac{\text{m}}{\text{s}^2}$.

Repeat this analysis when the same set blocks has its bottom-most block fastened to a rigid floor (shown to the right).

Result:

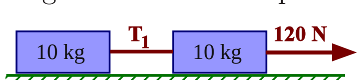
If **False** for left system:	If **False** for right system:
$T_1 =$ ____ N $\quad T_2 =$ ____ N	$T_1 =$ ____ N $\quad T_2 =$ ____ N



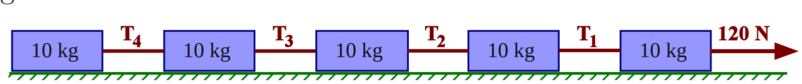
$T_1 = T_2 = T_3$
True/False

14.3 FE/EIT Review – Tension in horizontal ropes (free-body diagrams). $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$

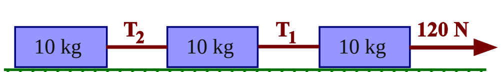
The following figures show rigid blocks, each of mass 10 kg, that are in contact with a smooth (**frictionless**) flat horizontal Newtonian reference frame. Each set of interconnected blocks is pulled horizontally-right with 120 Newtons. Using ***free-body diagrams***, determine the tension T_i ($i = 1, 2, \ldots$) in each light inextensible ropes connecting the blocks.



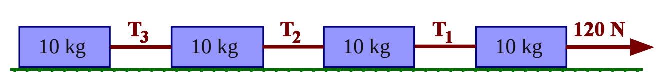
$T_1 =$ ____ Newtons



$T_4 =$ ____ N $\quad T_3 =$ ____ N $\quad T_2 =$ ____ N $\quad T_1 =$ ____ N



$T_2 =$ ____ N $\quad T_1 =$ ____ N



$T_3 =$ ____ N $\quad T_2 =$ ____ N $\quad T_1 =$ ____ N

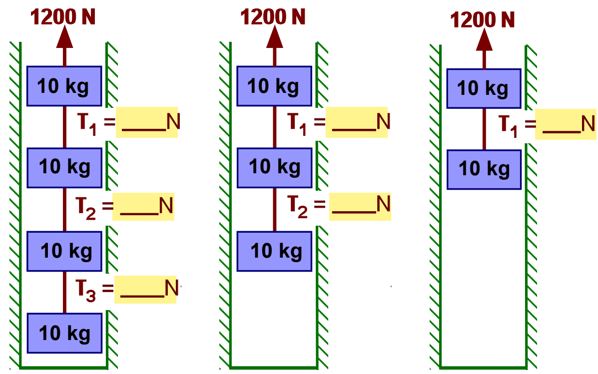
Hint: Consider ***MG road-maps*** and first calculate system center of mass acceleration, then T_1, then T_2, etc.

14.4 FE/EIT Review – Tension in vertical ropes (free-body diagrams). $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$

The figures to the right show 10 kg rigid blocks that are in smooth vertical slots on Earth (a Newtonian reference frame) whose gravity $g \approx 10\ \frac{\text{m}}{\text{s}^2}$.

Each set of interconnected blocks is pulled vertically-upward with 1200 Newtons.

Using ***free-body diagrams***,[a] determine the tension T_i $(i = 1, 2, 3)$ in the light inextensible ropes connecting the blocks.



[a] The **choice** of ***free-body diagrams*** is non-unique. Certain **choices** are computationally advantageous. ***Free-body diagrams*** (also called ***free-system diagrams***) may be a collection of bodies/objects (not just one body).

Using **your *free-body diagrams*** for the left-most set of blocks, cast your equations into matrix form.
Note: The result depends on your **choice** of ***free-body diagrams***. The results shown right used ***MG road-maps***.

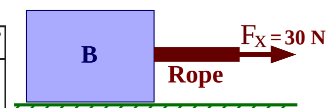
$$\begin{bmatrix} 40 & 0 & 0 & 0 \\ 30 & -1 & 0 & 0 \\ 20 & 0 & -1 & 0 \\ 10 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \ddot{y} \\ T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 800 \\ -300 \\ -200 \\ -100 \end{bmatrix}$$

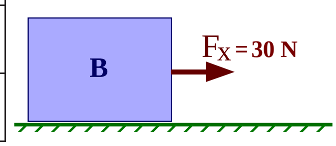
14.5 † Force transmitted through a rope or spring (free-body diagrams). $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$

The following figure shows an inextensible rope attached to a metallic particle B that is in contact with a **rough** flat horizontal magnetic table (a Newtonian reference frame). A horizontal force with measure $F_x = 30$ Newtons is applied to the distal end of the rope. **Ignore gravity** in this analysis.

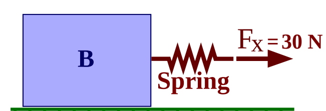
For each analysis, below decide whether it makes a difference to the block's motion or forces if F_x acts through the rope (top-right figure) or directly on B (bottom-right figure). **CIRCLE** Yes or No.

Mass of rope	Static/dynamic analysis	Makes a difference?
Massless	B and rope are stationary (statics)	**Yes/No**
Massive	B and rope are stationary (statics)	**Yes/No**
Massless	B and rope translate right at same constant speed	**Yes/No**
Massive	B and rope translate right at same constant speed	**Yes/No**
Massless	B and rope translate right at variable speeds	**Yes/No**
Massive	B and rope translate right at variable speeds	**Yes/No**





Shown right is a similar system, with the rope replaced by a **spring**.
Reconsider the previous question, except now **BOX** Yes or No.
Draw *free-body diagram(s)* and explain your spring answer.



Explain: ______.

Draw Free Body Diagrams (**FBD**) with:
- Physical object(s) in system
- Forces (contact & distance)

Knowing the block's mass is 5 kg, the rope's mass is 1 kg, $F_x = 30$ N, and the block is initially at rest, calculate the magnitude of the block's velocity relative to the table at $t = 4$ seconds.

Result: $\left|\vec{\mathbf{v}}(t=4)\right| = $ ____ $\frac{\text{m}}{\text{s}}$ (For this result only, assume the surface is frictionless).