Optional: Homework 16. Chapters 18, 21.

Show work – except for ♣ fill-in-blanks. **Forces, force models, and statics.**

16.1 ♣ Two types of forces in a *free-body diagram (FBD)* (Section 18.6).

From an engineering/FBD perspective, the two types of forces are contact and distance.

16.2 ♣ What is 1 Newton and 1 lb_f? (Section 21.1).

1 Newton is defined as (circle all that apply)	[✓ $1\ \frac{\text{kg m}}{\text{s}^2}$]	$9.81\ \frac{\text{kg m}}{\text{s}^2}$	$32.2\ \frac{\text{kg m}}{\text{s}^2}$	None of these
1 lb_f is defined as or approximately equal to (circle all that apply)	$1\ \text{kg}\cdot\frac{\text{m}}{\text{s}^2}$	[✓ $1\ \text{slug}\cdot\frac{\text{ft}}{\text{s}^2}$]	$1\ \text{lb}_\text{m}\cdot\frac{\text{ft}}{\text{s}^2}$	
	$9.81\ \text{kg}\cdot\frac{\text{m}}{\text{s}^2}$	$9.81\ \text{slug}\cdot\frac{\text{ft}}{\text{s}^2}$	$9.81\ \text{lb}_\text{m}\cdot\frac{\text{ft}}{\text{s}^2}$	
	$32.2\ \text{kg}\cdot\frac{\text{m}}{\text{s}^2}$	$32.2\ \text{slug}\cdot\frac{\text{ft}}{\text{s}^2}$	[✓ $32.2\ \text{lb}_\text{m}\cdot\frac{\text{ft}}{\text{s}^2}$]	

Using the exact Section 21.1 NIST conversion factor for lbm to kg and the exact conversion factor $1\text{ inch} \triangleq 2.54\text{ cm}$, show how to calculate the conversion factor for lbf to Newton.

Result

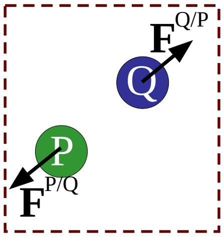
$$1\text{ lbf} \approx \frac{32.2\ \text{lbm ft}}{\text{s}^2} * \frac{0.45359237\ \text{kg}}{1\ \text{lbm}} * \frac{12\ \text{inch}}{1\ \text{ft}} * \frac{2.54\ \text{cm}}{1\ \text{inch}} * \frac{1\ \text{m}}{100\ \text{cm}} * \frac{1\ \text{N}}{1\ \text{kg m/s}^2} \approx 4.45\text{ N}$$

16.3 ♣ Force concepts (Section 18.3).

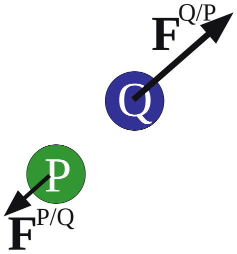
A force is a well-defined quantity:	**True/[✓ False]**
The resultant of a set of forces is a force.	**True/[✓ False]**
In the SI (metric) system, the units of force are:	Newtons
In the SI (metric) system, the units of impulse are:	Newton ∗ sec
$\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ is violated if a non-zero forces exists without the presence of a massive object	**[✓ True]/False**
$\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ is violated if mass exists without the presence of force	**True/[✓ False]**

16.4 ♣ Law of action/reaction (Section 18.1).

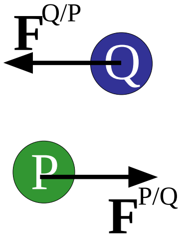
Circle the forces that obey the ***law of action/reaction***. **Explain** why each pair obeys/disobeys.



Forces have equal magnitude, opposite directions, and directed towards each other.



Force magnitudes are ***unequal***.



Forces are ***not*** directed along line $\overline{PQ}$ (different lines of action).

16.5 ♣ Coulomb's friction law (Section 21.6).

The range for Coulomb's ***coefficient of static friction*** is always $0 \leq \mu_s \leq 1$. **True/[✓ False]**.

Coulomb's friction law is **exact/accurate** (< 1% error)/**[✓ approximate]/completely wrong.**

16.6 ♣ Force and mass concepts (Section 21.1).

The center of mass and center of gravity may be different points.	**[✓ True]/False**
The center of gravity of a rigid body always exists.	**True/[✓ False]**

16.7 ♣ Convert U.S. units of psi to SI units of Pascals and estimate stress (Section 21.1).

Use <u>**each**</u> conversion factors (number) below at <u>**least once**</u> (and others you should know) to convert units of 1 inch to $\boldsymbol{x}$ meters, 1 lbf (pound-force) to $\boldsymbol{y}$ Newtons, and 1 psi ($\frac{\text{lbf}}{\text{inch}^2}$) to $\boldsymbol{z}$ Pascals ($\frac{\text{N}}{\text{m}^2}$).
Express your results for $\boldsymbol{z}$ in terms of the intermediate conversion factors $\boldsymbol{x}$ and $\boldsymbol{y}$.

$$1 \text{ inch} \triangleq 2.54 \text{ cm} \qquad 1 \text{ kg} \approx 2.2 \text{ lbm} \qquad 1 \text{ g}_{\text{Earth}} \approx 32.2 \tfrac{\text{ft}}{\text{s}^2}$$

Conversion	Equation (with numbers and units) for calculating result.	
$1 \text{ in} = \boldsymbol{x} \text{ m}$	$\boldsymbol{x} = 1 \text{ in} \, * \, \frac{2.54 \text{ cm}}{1 \text{ in}} \, * \, \frac{1 \text{ m}}{100 \text{ cm}} \approx 0.0254$	**Example of how to do units conversions**
$1 \text{ lbf} = \boldsymbol{y} \text{ N}$	$\boldsymbol{y} = 1 \text{ lbf} \, * \, \frac{1 \text{ lbm}*32.2 \frac{\text{ft}}{\text{s}^2}}{1 \text{ lbf}} \, * \, \frac{1 \text{ kg}}{2.2 \text{ lbm}} \, * \, \frac{12 \text{ in}}{1 \text{ ft}} \, * \, \frac{2.54 \text{ cm}}{1 \text{ in}} \, * \, \frac{1 \text{ m}}{100 \text{ cm}} \, * \, \frac{1 \text{ N}}{1 \text{ kg} \frac{\text{m}}{\text{s}^2}} \approx 4.45$	
$1 \text{ psi} = \boldsymbol{z} \text{ Pa}$	$\boldsymbol{z} = \frac{1 \text{ lbf}}{1 \text{ in}^2} \, * \, \frac{\boldsymbol{y} \text{ N}}{\text{lbf}} \, * \, \left(\frac{1 \text{ in}}{\boldsymbol{x} \text{ m}}\right)^2 \, * \, \frac{1 \text{ Pa}}{1 \text{ N/m}^2} \approx 6894.757$	**Express $\boldsymbol{z}$ in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$**

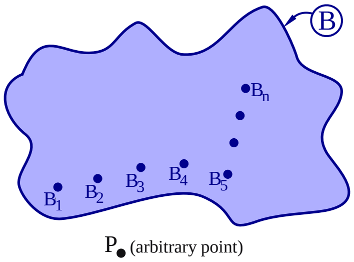
Engineering skill: Estimate/approximate order-of-magnitudes for mass, force, and stress.

Mass of a laptop computer	$\mathbf{0.04/0.4/\boxed{4}/40/400}$	lbm
Weight of a laptop computer	$\mathbf{0.04/0.4/\boxed{4}/40/400}$	lbf
Weight of a laptop computer	$\mathbf{0.02/0.2/2/\boxed{20}/200}$	Newtons
Gage pressure in a bike tire (remember pumping up a bike/car tire?)	$\mathbf{0.05/0.5/5/\boxed{50}/500}$	psi
Gage pressure in a bike tire	$\mathbf{0.03/0.3/3/30/\boxed{300}}$	kPa
Stress of a 200 lbf person on one flat sneaker on a wood floor	$\mathbf{0.05/0.5/\boxed{5}/50/500}$	psi
Stress of a 120 lbf person on one stiletto heel on a wood floor	$\mathbf{0.03/0.3/3/30/\boxed{300}}$	psi

16.8 ♣ Forces on an aluminum body (Sections 18.4, 18.2, and 19.4).

Consider 6.022×10^{23} molecules of aluminum (≈ 27 grams) which define a body B. The quantities under investigation (and related moments) are:

- $\vec{\mathbf{F}}^{B}$ the resultant of all forces on B
- $\vec{\mathbf{F}}^{B}_{\text{internal}}$ the resultant of all internal forces in B
- $\vec{\mathbf{F}}^{B}_{\text{external}}$ the resultant of all external forces on B
- $\vec{\mathbf{F}}^{B_1/B_2}$ the force on molecule B_1 from molecule B_2



Condition	Static equilibrium?	Circle the following statements that are always true.			
B is rigid	Yes	$\boxed{\vec{\mathbf{F}}^{B} = \vec{0}}$	$\boxed{\vec{\mathbf{F}}^{B}_{\text{internal}} = \vec{0}}$	$\boxed{\vec{\mathbf{F}}^{B}_{\text{external}} = \vec{0}}$	$\vec{\mathbf{F}}^{B_1/B_2} = \vec{0}$
		$\boxed{\vec{\mathbf{M}}^{B/P} = \vec{0}}$	$\boxed{\vec{\mathbf{M}}^{B/P}_{\text{internal}} = \vec{0}}$	$\boxed{\vec{\mathbf{M}}^{B/P}_{\text{external}} = \vec{0}}$	$\vec{\mathbf{M}}^{\vec{\mathbf{F}}^{B_2/B_1}/P} = \vec{0}$
B is flexible	Yes	$\boxed{\vec{\mathbf{F}}^{B} = \vec{0}}$	$\boxed{\vec{\mathbf{F}}^{B}_{\text{internal}} = \vec{0}}$	$\boxed{\vec{\mathbf{F}}^{B}_{\text{external}} = \vec{0}}$	$\vec{\mathbf{F}}^{B_1/B_2} = \vec{0}$
(a thin ruler)		$\boxed{\vec{\mathbf{M}}^{B/P} = \vec{0}}$	$\boxed{\vec{\mathbf{M}}^{B/P}_{\text{internal}} = \vec{0}}$	$\boxed{\vec{\mathbf{M}}^{B/P}_{\text{external}} = \vec{0}}$	$\vec{\mathbf{M}}^{\vec{\mathbf{F}}^{B_2/B_1}/P} = \vec{0}$
B is rigid	No	$\vec{\mathbf{F}}^{B} = \vec{0}$	$\boxed{\vec{\mathbf{F}}^{B}_{\text{internal}} = \vec{0}}$	$\vec{\mathbf{F}}^{B}_{\text{external}} = \vec{0}$	$\vec{\mathbf{F}}^{B_1/B_2} = \vec{0}$
		$\vec{\mathbf{M}}^{B/P} = \vec{0}$	$\boxed{\vec{\mathbf{M}}^{B/P}_{\text{internal}} = \vec{0}}$	$\vec{\mathbf{M}}^{B/P}_{\text{external}} = \vec{0}$	$\vec{\mathbf{M}}^{\vec{\mathbf{F}}^{B_2/B_1}/P} = \vec{0}$
B is molten	No	$\vec{\mathbf{F}}^{B} = \vec{0}$	$\boxed{\vec{\mathbf{F}}^{B}_{\text{internal}} = \vec{0}}$	$\vec{\mathbf{F}}^{B}_{\text{external}} = \vec{0}$	$\vec{\mathbf{F}}^{B_1/B_2} = \vec{0}$
(liquid metal)		$\vec{\mathbf{M}}^{B/P} = \vec{0}$	$\boxed{\vec{\mathbf{M}}^{B/P}_{\text{internal}} = \vec{0}}$	$\vec{\mathbf{M}}^{B/P}_{\text{external}} = \vec{0}$	$\vec{\mathbf{M}}^{\vec{\mathbf{F}}^{B_2/B_1}/P} = \vec{0}$

16.9 ♣ Measurement accuracies of the Universal Gravitational Constant G (Section 21.3).

The gravitational force on a particle Q of mass m^Q from a particle P of mass m^P can be written in terms of Q's position vector from P as

$$\boxed{\vec{\mathbf{F}}^{Q/P} \underset{(21.3)}{=} \frac{-G\, m^P\, m^Q}{\left|\vec{\mathbf{r}}^{Q/P}\right|^3}\, \vec{\mathbf{r}}^{Q/P}}$$

What are the dimensions of G (in terms of mass, length and time) and the SI units of G?

Result: Dimensions of G: (in terms of mass, length, time) $\dfrac{\text{length}^3}{\text{mass} * \text{time}^2}$ Units of G: (in terms of kg, m, s) $\dfrac{\text{m}^3}{\text{kg} * \text{s}^2}$

The number of significant digits the first measurement of the "***Universal Gravitational Constant***" G in 1798 by Cavendish and recent (year 2000$^+$) experiments of G are estimated to be accurate to:

Experiments in 1798	1	[✓ 2]	5	7	infinite
Experiments in 2000$^+$	1	[✓ 2]	5	7	infinite

Note: Section 21.3 discusses experiments involving G.

16.10 ♣ According to Newton's laws (theories), is the universe static? (Sections 18.2 and 19.3)

Consider the system S consisting of all matter, life, forces, etc.[a]

- Newton's laws dictates the resultant of all forces on S is $\vec{\mathbf{0}}$, i.e., $\vec{\mathbf{F}}^S = \vec{\mathbf{0}}$. [✓ **True**]/**False**.
- The moment of all forces on S about any point P is $\vec{\mathbf{0}}$, i.e., $\vec{\mathbf{M}}^{S/P} = \vec{\mathbf{0}}$. [✓ **True**]/**False**.
- Newton's laws dictates the universe is in **static equilibrium**. [✓ **True**]/**False**.
- Newton's **1**st/**2**nd/[✓ **3**rd] law explains my answer? (circle one)

Why: Since all forces are **inside** the system and the law of action/reaction states each force has an equal and opposite force, the sum (resultant) of the forces must be $\vec{\mathbf{0}}$.

[a] Carbon 12 has $\approx 6.02 \times 10^{23}$ molecules. The universe has $\approx 10^{80}$ molecules.

16.11 ♣ Free-Body Diagrams for 3D three-balloon system.

The picture below shows a cup hanging by strings from three motionless helium-filled balloons. Draw ***FBDs*** of each system for the given assumptions.

Helium is lighter than air.

FBD of cup:	FBD of left string:	FBD of left balloon:
Assumptions in FBD Only relevant forces are Earth's uniform gravity and contact forces from strings (no wind/air resistance, electromagnetic forces, etc.)	**Assumptions in FBD** String is inextensible. Only relevant forces are tension in string (gravity/other forces are negligible compared to tension.)	**Assumptions in FBD** Relevant forces are gravity from Earth, fluid pressure from air (buoyancy), and contact from strings and other balloons.