20.1 ♣ FE/EIT – Bungee jumper conservation of energy $K + U = \text{constant}$

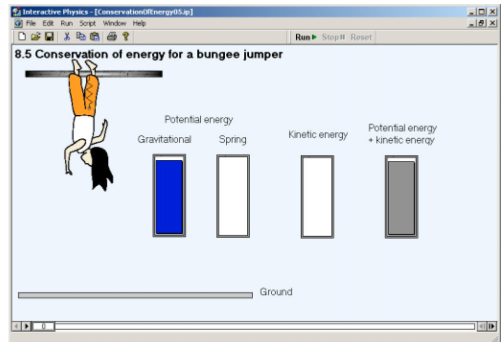
In an ideal energy situation (called ***conservation of energy***), potential energy U is converted to kinetic energy K and vice-versa without losing energy to sound, heat, etc. For example, the following bungee jumper is at **rest** on a platform before a jump. As she starts her jump at height $h = 0$ and falls towards the river, her gravitational potential energy U_{gravity} is converted into spring potential energy U_{spring} and kinetic energy K. At the bottom of her bounce, all her U_{gravity} and K have been converted to U_{spring}. As she starts her bounce back up, her U_{spring} converts to U_{gravity} and K. At the end of her upward bounce, she returns to height $h = 0$ and all energy is converted back to U_{gravity}.

$$U_{\text{gravity}} = m\,g\,h$$
$$U_{\text{spring}} = \tfrac{1}{2}\,k\,s^2$$
$$K = \tfrac{1}{2}\,m\,v^2$$

Bungee jumper's mass	m
Earth's gravitational acceleration	g
Bungee jumper's height	h
Spring constant (linear spring)	k
Spring stretch ($s = h$)	s
Bungee jumper's speed-squared	v^2

$$K + U = \text{constant}$$

Conservation of mechanical energy



(a) The gravitational potential energy is largest when the jumper is at the **top/middle/bottom,** and smallest when she is at the **top/middle/bottom** .

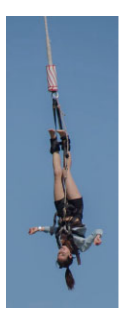
(b) When the bungee jumper is at the top, there is no stretch in the bungee cord. Therefore, the spring potential energy is **smallest/largest** . At the bottom, the bungee cord is highly stretched, and the spring potential energy is **smallest/largest** .

(c) The kinetic energy seems to be highest when the jumper is at the **top/middle/bottom** of a bounce. At this point, her speed is **smallest/largest** .

(d) The sum of potential energy and kinetic energy **increases/decreases/remains the same** .

(e) Complete the following table. Each row represents a different bungee jumper height.

U_{gravity} (Joules)	U_{spring} (Joules)	Kinetic energy K (Joules)	$K + U_{\text{gravity}} + U_{\text{spring}}$ (Joules)	Bungee jumper height h (meters)
1000	0	0		**top/bottom/in between**
356	415		1000	**top/bottom/in between**
	1000	0	1000	**top/bottom/in between**
276		200		**top/bottom/in between**



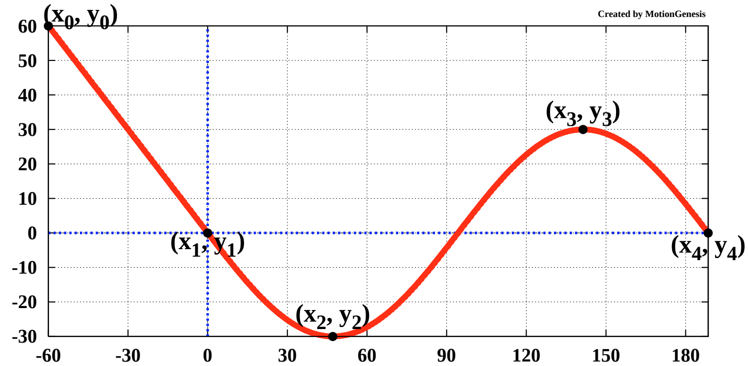
20.2 FE/EIT – Rollercoaster: $\vec{F} = m\,\vec{a}$ and conservation of mechanical energy

The shape of a roller-coaster track is approximated by a two-part function $y(x)$.

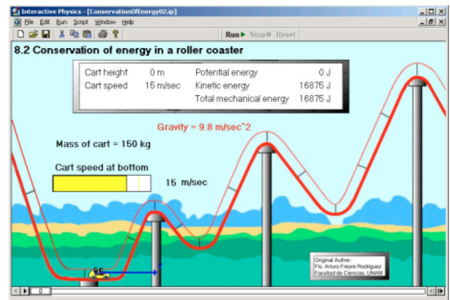
The 1^{st} part is a straight line connecting (x= -60, y= 60) to (0, 0).

The 2^{nd} part is a sine curve with amplitude 30 m and wave-length ($60\,\pi \approx 188.5$) m.

$$y(x) = \begin{cases} -x & x < 0 \\ -30\,\sin(\frac{x}{30}) & x \geq 0 \end{cases}$$



A roller-coaster car is released from **rest** at (x= -60, y= 60). The track is smooth (frictionless), Earth's gravitational acceleration $g \approx 10\ \frac{m}{s^2}$, and the roller-coaster car with occupants is modeled as a particle with mass $m = 100$ kg. For each location $x_1 \ldots x_4$, calculate the roller-coaster speed $v_1 \ldots v_4$ (3^+ digits). Determine the time t_1 when the roller-coaster car is at x_1.



Location		Frictionless	
x (m)	y (m)	Speed (m/s)	time t (sec)
x_0 = -60	y_0 = 60	$v_0 = 0$ (given)	t_0 = 0
x_1 = 0	y_1 = 0	v_1 =	$t_1 \approx$
$x_2 = 15\,\pi$	y_2 = -30	v_2 =	$t_2 \approx$ 6.4
$x_3 = 45\,\pi$	y_3 = 30	v_3 =	$t_3 \approx$ 9.8
$x_4 = 60\,\pi$	y_4 = 0	$v_4 = \sqrt{1200} \approx 34.6$	$t_4 \approx$ 11.8

† **Optional:** Determine times $t_2 \ldots t_4$. Hint: Run a simulation and output $\frac{x}{\pi}$ and t + 4.898979.

†† **Optional:** Determine the same quantities using a coefficient of kinetic friction $\mu_k = 0.1$.

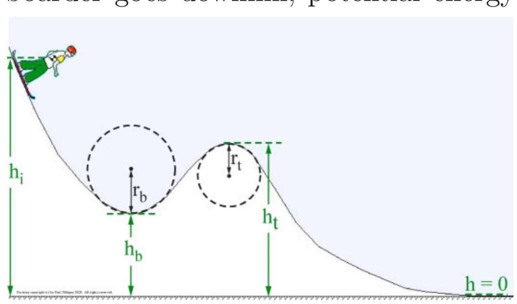
20.3 FE/EIT – Snowboarder: Conservation of energy to normal forces

In ideal situations, potential energy can be converted to kinetic energy and vice versa [i.e., kinetic energy plus potential energy is ***conserved*** (constant)]. For example, a snowboarder gains potential energy when chair-lifted to the top of the mountain. As the snowboarder goes downhill, potential energy decreases and speed and kinetic energy increases.

$$U_{\text{gravity}} = m\ g\ h \qquad K = \tfrac{1}{2}\ m\ v^2$$

$$\Delta K + \Delta U = 0$$

Conservation of mechanical energy



Model the snowboarder as a particle of mass m that starts from **rest** at initial height h_i above the bottom of the hill. Assume there is negligible air-resistance, friction, etc.

(a) Express the snowboarder's speed v_f at a height h_f above the bottom of the hill (in terms of g, h_i, h_f). Using $g \approx 10\ \frac{m}{s^2}$, calculate the snowboarder's speed when boarding down a 30° slope for a distance 160 m (along the slope). Report results in both $\frac{m}{s}$ and mph ($1\ \frac{m}{s} \approx 2.2$ mph).

Result: v_f =

Distance along slope	Speed at end	Realistic?
160 m	$\frac{m}{s} \approx$ mph	

(b) Referring to the snowboarding picture, the bottom part of the first dip is a height h_b above the bottom of the hill and is modeled as a downward semi-circular shape of radius r_b. Determine