

13.1 ♣ Concepts: Define and draw the moment of a force

Write the *definition* for the moment of force  $\vec{F}^Q$  applied to point  $Q$  about point  $O$ . Draw a sketch with *each* part of your definition clearly labeled.

Result:

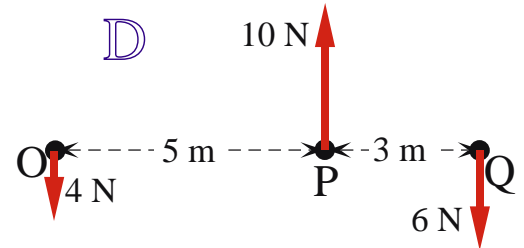
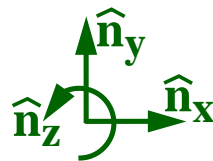
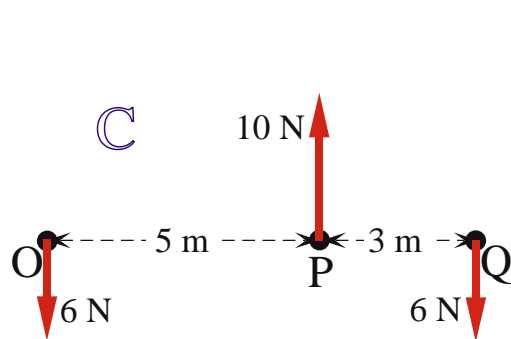
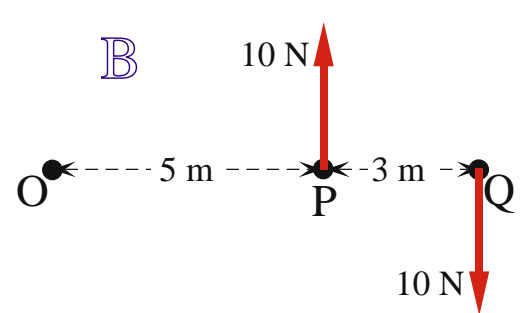
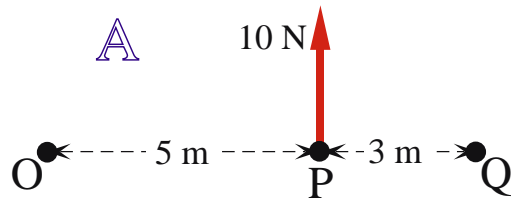
$$\vec{M}^{\vec{F}^Q/O} \triangleq \text{[ ]} \times \text{[ ]}$$



13.2 ♣ Moment vs. torque (refer to Section 17.5)

Consider the various sets  $S$  of forces, their resultants  $\vec{F}^S$ , and moments about points  $O$ ,  $P$ , and  $Q$ . This example shows how to easily determine whether a moment is a torque.<sup>1</sup>

$S$	$\vec{F}^S$	$\vec{M}^{S/O}$	$\vec{M}^{S/P}$	$\vec{M}^{S/Q}$	$\vec{M}^{S/O} \stackrel{?}{=} \vec{M}^{S/P} \stackrel{?}{=} \vec{M}^{S/Q}$	Moment is torque?
A	$10 \hat{n}_y$	$50 \hat{n}_z$	$\vec{0}$	[ ]	Yes/No	Yes/No
B	[ ]	[ ]	[ ]	[ ]	Yes/No	Yes/No
C	[ ]	[ ]	[ ]	[ ]	Yes/No	Yes/No
D	[ ]	[ ]	[ ]	[ ]	Yes/No	Yes/No



13.3 ♣ Moment and torque concepts

75% All torques are moments.

True/False

61% All moments are torques.

True/False

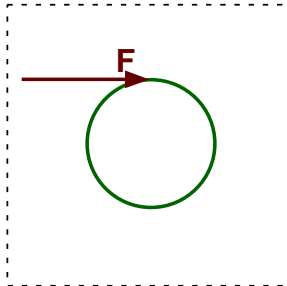
61% The moment of a couple about a point  $O$  is equal to the moment of the couple about any other point  $P$  True/False

<sup>1</sup>Since  $\vec{T}^S \triangleq \vec{M}^{S/O}$  if  $\vec{F}^S = \vec{0}$  (point  $O$  is *any* point), the *moment* is a *torque* if  $\vec{F}^S = \vec{0}$  (it is that simple).

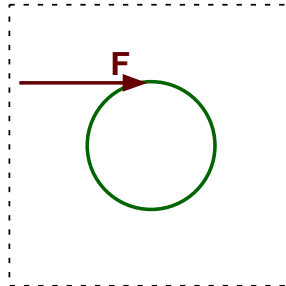
### 13.4 ♣ Drawing couples

Each figure below shows a single force  $\vec{F}$  applied tangentially to a point on the periphery of a circle. Complete each figure by drawing couples consisting of **2**, **3**, and **4** forces, respectively, so:

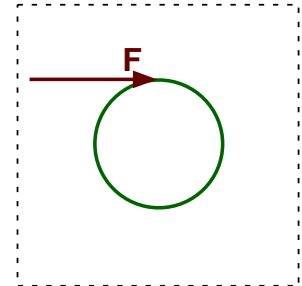
- Each force has magnitude  $|\vec{F}|$  and is applied at distinct points on the circle's periphery
- Each force is directed **tangent** to the circle's periphery
- The set of forces create a couple with non-zero torque



Couple with **two** forces



Couple with **three** forces

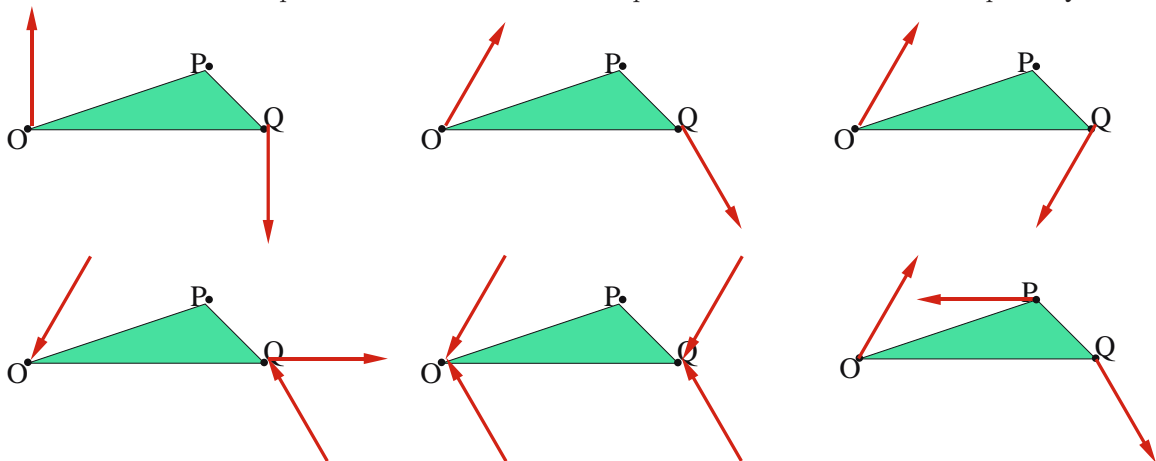


Couple with **four** forces

### 13.5 ♣ Moments of forces about various points

Consider the six figures below, each which contain a set of forces. Circle the figure(s) in which the moment of its set of forces about points  $O$ ,  $P$ , and  $Q$  all are equal, i.e.,

$$\text{Moment around point } O = \text{Moment around point } P = \text{Moment around point } Q$$



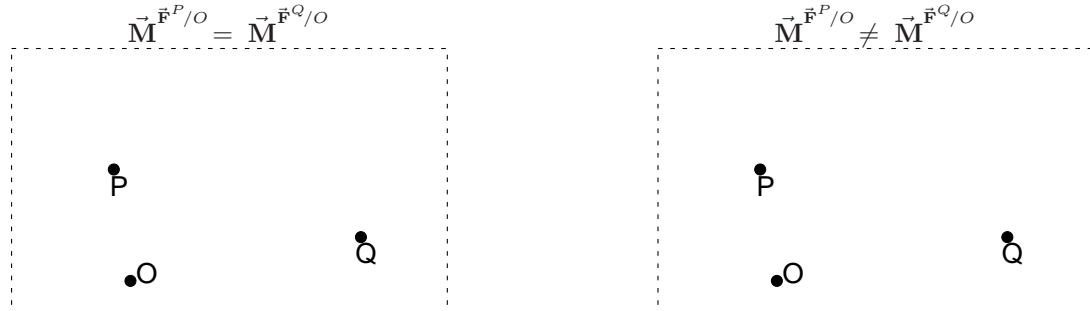
Note: All forces have the same magnitude. Forces that are not horizontal or vertical are  $30^\circ$  from vertical.

### 13.6 ♣ Forces, moments, and lines of action

**Draw** a non-zero force  $\vec{F}^P$  on point  $P$  and a non-zero force  $\vec{F}^Q$  on point  $Q$  so:

- $\vec{F}^P = \vec{F}^Q$  (force on  $P$  has the same magnitude and direction as the force on  $Q$ )
- $\vec{M}^{\vec{F}^P/O}$  (moment of  $\vec{F}^P$  about point  $O$ ) is **equal** to  $\vec{M}^{\vec{F}^Q/O}$  (moment of  $\vec{F}^Q$  about  $O$ ).

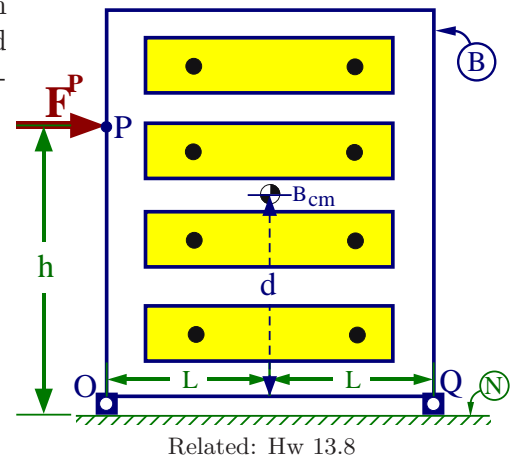
Repeat on the right-figure below, except ensure  $\vec{F}^P$  and  $\vec{F}^Q$  produce **unequal** moments about  $O$ .



### 13.7 FE/EIT Review – Bureau tipping and static friction.

The figure to the right shows a rigid uniform-density bureau  $B$  in contact with a **rough** flat horizontal surface  $N$  at points  $O$  and  $Q$  of  $B$ . A person (not shown) is pushing the bureau horizontally-right with a force of measure  $F^P$  applied to point  $P$  of  $B$ .

Description	Symbol
Mass of bureau	$m$
Earth's gravitational acceleration	$g$
Half-width of bureau	$L$
Distance between points $O$ and $P$	$h$
Distance between $B_{cm}$ and line $OQ$	$d$
Measure of normal force on $P$ from person	$F^P$
Measure of normal force on $O$ from inclined plane	$F_y^O$
Measure of normal force on $Q$ from inclined plane	$F_y^Q$



Related: Hw 13.8

Regard this system as planar and in **static equilibrium**.

This question investigates the role of **static friction** on bureau tipping. Answer the following questions in terms of symbols in the table and  $\mu_s$  (the **coefficient of static friction** between  $B$  and  $N$ ).

- **Before forming any equations, draw** a **free-body diagram (FBD)** of  $B$  and use your intuition (guess) whether or not the **start** of the bureau tipping depends on (circle all that apply).

$h \quad L \quad d \quad m \quad g \quad \mu_s$

- One way for the bureau to start tipping is for it to lose contact with  $N$  at point  $O$ . At this instant, the normal and friction forces at  $O$  are  $\vec{0}$  [ $N$  can push upward (not pull downward) on  $B$ ]. In this context, determine the minimum value of  $F^P$  to tip  $B$  clockwise. Next, determine  $h_{\text{slide}}$ , the maximum height where a sufficient push makes the bureau **start** to slide (not tip).

**Result:** Tip minimum push:  $F_{\text{minimum}}^P = \frac{\quad}{\quad}$  Slide max height:  $h_{\text{slide}} = \frac{\quad}{\quad}$

- When  $\mu_s \approx 0$ ,  $h_{\text{slide}}$  is **much smaller/smaller/equal to/larger/much larger** than  $L$ .
- In view of this **static analysis**, the **start** of bureau tipping depends on (circle all that apply)

$h \quad L \quad d \quad m \quad g \quad \mu_s$