12.1 ♣ Sort from smallest mass unit to largest mass unit. (see Section 13.1)

1 oz_m 1 g 1 metric ton 1 kg 1 mg 1 U.S. ton 1 slug 1 lb_m

12.2 ♣ Concepts: What objects have kinetic energy or linear momentum?

${}^{N}K^{S}$, the ***kinetic energy*** of an object S in a reference frame N is to be determined.
Objects S that can have a non-zero kinetic energy are (circle ***all*** appropriate objects):

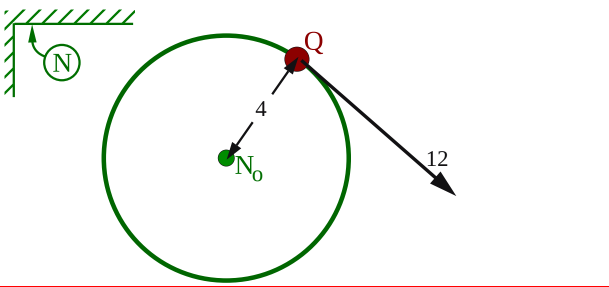
Real number	Matrix	Set of points	Mass center of a rigid body
Vector	Point	Reference frame	Flexible body
3D orthogonal unit basis	Particle	Rigid body	System of particles and bodies

Repeat for ${}^{N}\vec{\mathrm{L}}^{S}$, the ***linear momentum*** of object S in reference frame N [box appropriate objects].

12.3 ♣ Particle angular momentum concepts.

The following figures show a particle Q of mass 1 kg moving in a **plane** N. Point N_o is fixed in N. The figure on the left shows Q moving clockwise with speed 12 on a circle of radius 4 that is centered at N_o. The figure on the right shows Q moving with a speed of 12 on a horizontal line that is 4 from N_o. **Box** the following true statements about Q's ***angular momentum*** in N.

Q's angular momentum about N_o is $\vec{\mathbf{0}}$.
Q's angular momentum about N_o is not $\vec{\mathbf{0}}$.
Q's angular momentum about N_o is $\vec{\infty}$.
Q's angular momentum about N_o does not exist.



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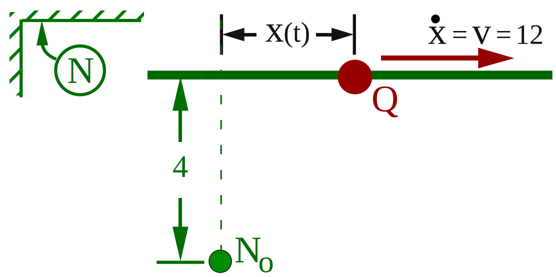
Q's angular momentum about N_o is $\vec{\infty}$.
Q's angular momentum about N_o does not exist.



12.4 ♣ Optional: Just for fun. Culture, religion, science and "mass". (Sections 13.9, 13.7, 13.6)

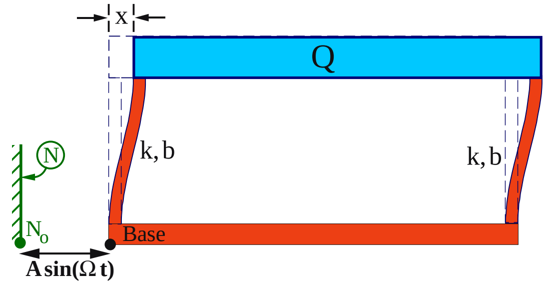
Etymology of "mass"	Fill-in the blank
The "m" in $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$.	
Jewish Passover flat bread/cracker.	
Greek for flat bread.	
Latin for lump of dough.	
Spanish for lump of dough.	
Catholics eat bread at this Sunday event.	
Approximate number of atoms in 12 grams of carbon-12.	6 x 10
Estimated number of atoms in the visible universe.	1 x 10
Sub-atomic particle responsible for mass in animals, vegetables, and minerals.	
Most expensive science project in history to find sub-atomic particle with mass.	
Possible Earth-fatal object created by aforementioned science project.	

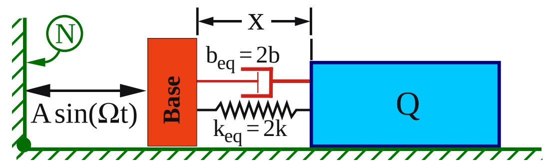
12.5 FE/EIT Review – Motion of a building in an earthquake.
$\vec{\mathbf{F}} \Rightarrow \vec{\mathbf{F}} = m\,\vec{\mathbf{a}} \Rightarrow \ddot{x}$

A building moves due to an earthquake. The horizontally-right displacement of the building's base B is **modeled** as $A\,\sin(\Omega\,t)$ where the constant A is the magnitude of the ground's horizontal displacement and the constant Ω is the earthquake's frequency.

The base motion causes the building's roof Q of mass m to displace horizontally by $x(t)$ from its base. The stiffness and material damping in each of the two columns that support the roof is modeled as a linear horizontal spring (k) and linear horizontal damper (b).

For this dynamic analysis, the system is modeled as shown right (with a spring of 0 natural length). It is helpful to introduce a horizontally-right unit vector $\widehat{\mathbf{n}}_{\mathrm{x}}$.





x is the $\widehat{\mathbf{n}}_{\mathrm{x}}$ measure of Q's position from its base B

(a) Draw Q's ***free-body diagram*** and determine the spring/damper force on Q.
Result: $\vec{\mathbf{F}}_{\text{Spring/Damper}} \underset{\textbf{FBD}}{=} (-b_{\text{eq}}\,\dot{x} \;+\; -k_{\text{eq}}\,x)\,\widehat{\mathbf{n}}_{\mathrm{x}}$

FBD of Q

(b) Form the relevant acceleration for $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ (e.g., differentiate the relevant position vector/velocity). Next, dot-product $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ with $\widehat{\mathbf{n}}_{\mathrm{x}}$ to write a differential equation governing $x(t)$.
Result: [**Optional:** Verify results via Kane and/or Lagrange methods in Chapters 26, 27.]

$$\vec{\mathbf{a}} = [\quad - \quad]\,\widehat{\mathbf{n}}_{\mathrm{x}} \qquad \Rightarrow \qquad m\,\ddot{x} \;+\; b_{\text{eq}}\,\dot{x} \;+\; k_{\text{eq}}\,x \;=\; m\,A\,\Omega^2\,\sin(\Omega\,t)$$

(c) Graphed right is $x(t)$ when:

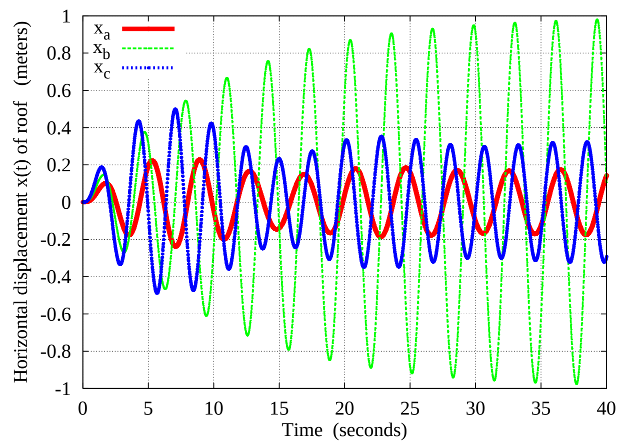
(a) $\Omega \;=\; 0.8\sqrt{\frac{k_{\text{eq}}}{m}} \;=\; 0.8\,\omega_n \qquad x_a(t)$

(b) $\Omega \;=\; 1.0\sqrt{\frac{k_{\text{eq}}}{m}} \;=\; 1.0\,\omega_n \qquad x_b(t)$

(c) $\Omega \;=\; 1.2\sqrt{\frac{k_{\text{eq}}}{m}} \;=\; 1.2\,\omega_n \qquad x_c(t)$

Circle the Ω that corresponds to the largest ***steady-state*** amplitude for $x(t)$.

Note: These graphs use $m \;=\; 5000$ kg, $b_{\text{eq}} \;=\; 1000\,\frac{\text{N*sec}}{\text{m}}$, $k_{\text{eq}} \;=\; 20000\,\frac{\text{N}}{\text{m}}$, $A \;=\; 0.1$ m.



(d) **Physics** ($\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$) gives the previous (boxed) equation in terms of positive constants m, b_{eq}, k_{eq}. However, its **mathematics** is easier if that equation is rewritten (rearrange by dividing by m) in terms of the positive constants ζ and ω_n as shown in the boxed-equation below. Determine the building's ***natural frequency*** ω_n and ***damping ratio*** ζ in terms of m, b_{eq}, k_{eq}.
Result:

$$\ddot{x} \;+\; 2\,\zeta\,\omega_n\,\dot{x} \;+\; \omega_n^2\,x \;=\; A\,\Omega^2\,\sin(\Omega\,t) \qquad \Rightarrow \qquad \omega_n = \sqrt{\frac{\quad}{\quad}} \qquad \zeta = \frac{\quad}{2\sqrt{\quad\;\quad}}$$

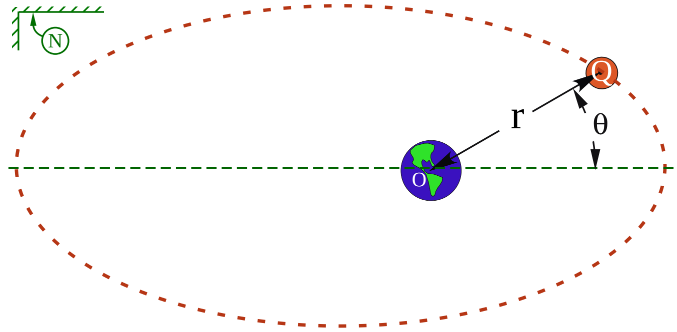
(e) When $\zeta \;=\; 0$, the building vibrates at ω_n. When $0 \le \zeta \le 1$ (common for many structures), the building vibrates at a ***damped natural frequency*** $\omega_d \triangleq \omega_n\,\sqrt{1-\zeta^2}$.
In general, damping slows things down and makes $\omega_d \;<\; \omega_n$. **True/False**.

12.6 FE/EIT Review – Momentum, energy, and orbital mechanics. $\vec{\mathbf{F}} \;\Rightarrow\; \vec{\mathbf{F}} = m\,\vec{\mathbf{a}} \;\Rightarrow\; \ddot{r},\ \ddot{\theta}$

The following figure shows a satellite Q (modeled as a particle of mass m) in an elliptical orbit around Earth. Earth is modeled as a particle O fixed in a Newtonian reference frame N.

Draw right-handed orthogonal unit vectors $\widehat{\mathbf{n}}_x$, $\widehat{\mathbf{n}}_y$, $\widehat{\mathbf{n}}_z$ fixed in N with $\widehat{\mathbf{n}}_x$ horizontally-right and parallel to the ellipse's major diameter, $\widehat{\mathbf{n}}_y$ vertically-upward and parallel to the ellipse's minor diameter, and $\widehat{\mathbf{n}}_z$ perpendicular to the ellipse's plane.

Draw right-handed orthogonal unit vectors $\widehat{\mathbf{e}}_r$, $\widehat{\mathbf{e}}_\theta$, $\widehat{\mathbf{e}}_z$, with $\widehat{\mathbf{e}}_r$ from O to Q and $\widehat{\mathbf{e}}_z = \widehat{\mathbf{n}}_z$.



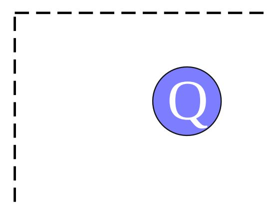
Quantity	Symbol	Type	Value
Universal gravitational constant	G	Constant	6.67×10^{-11} N∗m²/kg²
Mass of Earth O	m^E	Constant	5.97×10^{24} kg
Mass of Q	m	Constant	200 kg
Angle between line OQ and long axis of the ellipse	θ	Variable	Initial value
Distance between O and Q	r	Variable	Initial value

(a) Form Q's ***linear momentum***, ***angular momentum*** about O, and ***kinetic energy*** in N.
Result: (in terms of symbols in the previous table, their time-derivatives, and $\widehat{\mathbf{e}}_r$, $\widehat{\mathbf{e}}_\theta$, $\widehat{\mathbf{e}}_z$).

$${}^N\vec{\mathbf{L}}^Q \underset{(12.1)}{=} m(\boxed{} + \boxed{}) \qquad {}^N\vec{\mathbf{H}}^{Q/O} \underset{(12.3)}{=} \boxed{} \qquad {}^N K^Q \underset{(12.6)}{=} \frac{1}{2} m[\boxed{}^2 + (\boxed{})^2]$$

(b) **Draw** Q's free-body diagram and determine the resultant force on Q.
Dot $\vec{\mathbf{F}}^Q = m\,{}^N\vec{\mathbf{a}}^Q$ with "clever" unit vectors and solve for $\ddot{r}$ and $\ddot{\theta}$.
Result:

$$\vec{\mathbf{F}}^Q = \frac{\boxed{}}{\boxed{}}\,\widehat{\mathbf{e}}_r \qquad \ddot{r} = \boxed{} - \frac{\boxed{}}{r^2} \qquad \ddot{\theta} = \frac{\boxed{}}{r}$$



(c) Form an expression for Q's mechanical energy in N (sum of kinetic energy and potential energy U).
Result: ***Conservation of mechanical energy*** $\quad \text{KePe} = \frac{1}{2}\,m\left[\dot{r}^2 + (\dot{\theta}\,r)^2\right] - \frac{G\,m\,m^E}{r}$

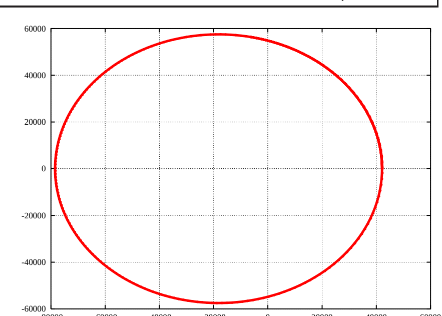
(d) Starting with the ***angular momentum principle*** in equation (22.4), form an expression that stays constant during Q's motion in N ***(conservation of angular momentum)***.
Result: $\quad H_{\text{Constant}} \underset{(22.9)}{=} \boxed{}$ (Section 22.7)

(e) **Draw** a line L tangent to the ellipse at Q and perpendicular to $\widehat{\mathbf{n}}_z$.

In general,	${}^N\vec{\mathbf{v}}^Q$ (Q's velocity in N) is parallel to $\widehat{\mathbf{e}}_\theta$.	**True/False**
In general,	$\widehat{\mathbf{e}}_\theta$ is parallel to L.	**True/False**
In general,	${}^N\vec{\mathbf{v}}^Q$ is parallel to L.	**True/False**

(f) Shown right is a plot of Q's orbital trajectory when the differential equations from part (6b) are solved with the initial values of part (6i). Query the Internet (or a textbook, instructor, colleague, etc.) for the next answers.

- Clearly mark Earth's location on the plot.
- The Earth is located at (circle one):
  the center of the ellipse/a focus of the ellipse.



(g) Form ***Lagrange's equations*** for generalized coordinates r and θ and solve for $\ddot{r}$ and $\ddot{\theta}$.
Result: Note: U is given above and $K = {}^N K^Q$. ***Lagrange's equations*** are described in Chapter 27.

$$-\frac{\partial U}{\partial r} \underset{(27.1)}{=} \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{r}}\right) - \frac{\partial K}{\partial r}$$
$$-\frac{\partial U}{\partial \theta} \underset{(27.1)}{=} \frac{d}{dt}\left(\frac{\partial K}{\partial \dot{\theta}}\right) - \frac{\partial K}{\partial \theta} \quad \Rightarrow \quad \ddot{r} = \boxed{} - \frac{\boxed{}}{r^2} \qquad \ddot{\theta} = \frac{\boxed{}}{r}$$

Just calculate!

Lagrange's equations result in the same $\ddot{r}$ and $\ddot{\theta}$ as part (6b). **True/False**.

(h) Form ***Kane's equation of motion*** for the generalized speeds $\dot{r}$ and $\dot{\theta}$ and solve for $\ddot{r}$ and $\ddot{\theta}$.
Result: Note: ***Kane's equations*** are described in Chapter 26.

$$\mathcal{F}_{\dot{r}} = -\frac{\partial U}{\partial r} \qquad {}^N\mathcal{F}^Q_{\dot{r}} = \frac{\partial\, {}^N\vec{\mathbf{v}}^Q}{\partial \dot{r}} \cdot m\, {}^N\vec{\mathbf{a}}^Q$$
$$\mathcal{F}_{\dot{\theta}} = -\frac{\partial U}{\partial \theta} \qquad {}^N\mathcal{F}^Q_{\dot{\theta}} = \frac{\partial\, {}^N\vec{\mathbf{v}}^Q}{\partial \dot{\theta}} \cdot m\, {}^N\vec{\mathbf{a}}^Q$$
$$\Rightarrow \quad \mathcal{F}_{\dot{r}} \underset{(26.1)}{=} {}^N\mathcal{F}^Q_{\dot{r}} \qquad \mathcal{F}_{\dot{\theta}} \underset{(26.1)}{=} {}^N\mathcal{F}^Q_{\dot{\theta}} \quad \Rightarrow \quad \ddot{r} = r\,\dot{\theta}^2 - \frac{G\, m^E}{r^2} \qquad \ddot{\theta} = \frac{-2\,\dot{\theta}\,\dot{r}}{r}$$

Just calculate!

Kane's equations result in the same $\ddot{r}$ and $\ddot{\theta}$ as part (6b). **True/False**.

(i) **Optional:** Use a computer to solve the differential equations for $\theta(t)$ and $r(t)$ with the initial values (i.e, values at $t = 0$) of θ, $\dot{\theta}$, r, and $\dot{r}$ shown below. Use a numerical integration step (`tStep`) of 0.2 hours. Plot y vs x for $0 \le t \le 48$ hours.
Result: Submit your computational files (e.g., MotionGenesis, MATLAB®) and plot of y vs. x.

Sample MotionGenesis commands

```
Constant  G = 6.67E-11 N*m^2/kg^2,  mE = 5.97E24 kg,  m = 200 kg
Variable  r'',  theta''
r'' = r * theta'^2  -  G * mE / r^2
theta'' = -2 * theta' * r' / r
Input  theta = 0 degrees,  theta' = 0.3 rad/hour,  r = 42108 km,  r' = 0 km/hour
Input  tFinal = 48 hours,  tStep = 0.2 hours,  absError = 1.0E-7
OutputPlot  x = r*cos(theta);  y = r*sin(theta)
Output  t hours,  theta degrees,  r km, x km, y km,  HConstant kg*m^2/sec,  KePe Joules
```

(j) **Optional:** Verify each value in the following chart.

Quantity	Value
$H_{constant}$	2.955×10^{13} kg∗m^2/sec
Sum of kinetic and potential energy ($KePe$)	-6.606×10^{8} Joules
Number of hours to complete one orbit	41 hours
Minimum distance between O and Q	42108 km
Maximum distance between O and Q	78500 km