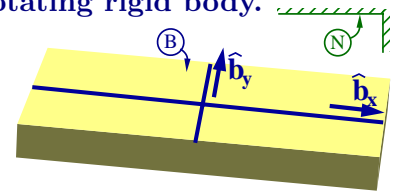


11.6 Calculate: Angular momentum and kinetic energy of a 3D rotating rigid body.

Consider a rigid body B rotating in a Newtonian reference frame N . Right-handed orthogonal unit vectors $\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$ are fixed in B and parallel to B 's principal inertia axes about B_{cm} (B 's center of mass).



Quantity	Symbol	Type
B 's moment of inertia about B_{cm} for $\hat{\mathbf{b}}_x$	I_{xx}	Constant
B 's moment of inertia about B_{cm} for $\hat{\mathbf{b}}_y$	I_{yy}	Constant
B 's moment of inertia about B_{cm} for $\hat{\mathbf{b}}_z$	I_{zz}	Constant
$\hat{\mathbf{b}}_x$ measure of ${}^N\vec{\omega}^B$	ω_x	Variable
$\hat{\mathbf{b}}_y$ measure of ${}^N\vec{\omega}^B$	ω_y	Variable
$\hat{\mathbf{b}}_z$ measure of ${}^N\vec{\omega}^B$	ω_z	Variable

$${}^N\vec{\omega}^B = \omega_x \hat{\mathbf{b}}_x + \omega_y \hat{\mathbf{b}}_y + \omega_z \hat{\mathbf{b}}_z$$

$$\overset{B/B_{cm}}{\mathbf{I}} = I_{xx} \hat{\mathbf{b}}_x \hat{\mathbf{b}}_x + I_{yy} \hat{\mathbf{b}}_y \hat{\mathbf{b}}_y + I_{zz} \hat{\mathbf{b}}_z \hat{\mathbf{b}}_z$$

$$= \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}_{\hat{\mathbf{b}}_{xyz}}$$

Calculate B 's angular momentum about B_{cm} in N and B 's rotational kinetic energy in N .

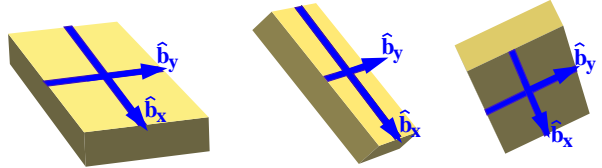
Result: ${}^N\vec{H}^{B/B_{cm}} = I_{xx} \omega_x \hat{\mathbf{b}}_x + I_{yy} \omega_y \hat{\mathbf{b}}_y + \text{[yellow box]} \hat{\mathbf{b}}_z$ ${}^N K_{\text{rotation}}^B = \frac{1}{2} (I_{xx} \omega_x^2 + I_{yy} \omega_y^2 + \text{[yellow box]})$

11.7 3D spinning rigid body: Guess and check conservation of energy/angular momentum.

This problem refers to Homeworks 6.18 and 11.6.

$$\vec{M} = \frac{N d \vec{H}}{dt} \quad (20.4)$$

$$\begin{aligned} \dot{\omega}_x &= [(I_{yy} - I_{zz}) \omega_z \omega_y] / I_{xx} \\ \dot{\omega}_y &= [(I_{zz} - I_{xx}) \omega_x \omega_z] / I_{yy} \\ \dot{\omega}_z &= [(I_{xx} - I_{yy}) \omega_y \omega_x] / I_{zz} \end{aligned}$$



- Using your intuition and/or **conservation of angular momentum/mechanical energy** (Sections 20.9 and 23.2), circle the quantities below that you **guess** remain constant (are “conserved”).
- Next, solve the ODEs in Hw 6.18 for $0 \leq t \leq 4$ with initial values $\omega_x = 7, \omega_y = 0.2, \omega_z = 0.2$ (use MotionGenesis, MATLAB®, or ...). Output $t, \omega_x, \omega_y, \omega_z, H_x, H_y, H_z, H_{mag} \triangleq |\vec{H}|$, and K .
- Circle the quantities that remain constant (are “conserved”) while solving the ODEs.

ω_x	ω_y	ω_z	K
$H_x = \vec{H} \cdot \hat{\mathbf{b}}_x$	$H_y = \vec{H} \cdot \hat{\mathbf{b}}_y$	$H_z = \vec{H} \cdot \hat{\mathbf{b}}_z$	$ \vec{H} $

Hint: To determine if a quantity such as ω_x is constant during numerical integration, look at the numerical values of ω_x or **zoom** into the plot of ω_x vs. t and check if ω_x is constant to within a reasonable multiplier (e.g., 10) of numerical integrator accuracy. In MotionGenesis and MATLAB®, the default numerical integrator accuracy is $\approx 1 \times 10^{-6}$.

Solution at www.MotionGenesis.com \Rightarrow [Get Started](#) \Rightarrow Solving 1st-order ODEs.

Add the line: $H_x = I_{xx} \omega_x; H_y = I_{yy} \omega_y; H_z = I_{zz} \omega_z; H_{mag} = \text{sqrt}(H_x^2 + H_y^2 + H_z^2)$, etc.

Add the line: Output t sec, H_x kg*m²/sec, H_y kg*m²/sec, H_z kg*m²/sec, H_{mag} kg*m²/sec, etc.

View/plot the numerical results (numbers) and determine if they stay constant to within $\approx 1 \times 10^{-5}$.

Optional: Simulate delayed 3D spin instability for wingnut or T-handle with: $I_{xx} < I_{yy} < I_{zz}$, spin is initially mostly about $\hat{\mathbf{b}}_y$ (intermediate moment-of-inertia axis), and $(I_{yy} - I_{xx})(I_{zz} - I_{yy})$ is small, e.g., by using $\omega_x(0) = 0.2 \frac{\text{rad}}{\text{sec}}, \omega_y(0) = 7.0 \frac{\text{rad}}{\text{sec}}, \omega_z(0) = 0.2 \frac{\text{rad}}{\text{sec}}, I_{xx} = 1.9 \text{ kg m}^2, I_{yy} = 2.0 \text{ kg m}^2, I_{zz} = 3.0 \text{ kg m}^2$.

Video and Dzhanibekov analysis at www.MotionGenesis.com \Rightarrow [Get Started](#) \Rightarrow Spin stability

