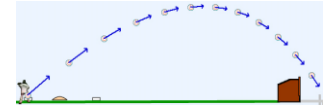


### 21.1.1 MG road-map: Projectile motion (2D)

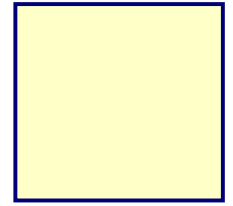
A baseball (particle  $Q$ ) flies over Earth  $N$  (a Newtonian reference frame). Aerodynamic forces on the baseball are modeled as  $-b\vec{v}$  ( $\vec{v}$  is  $Q$ 's velocity in  $N$ ).

$\hat{n}_x$  is horizontally-right,  $\hat{n}_y$  is vertically-upward, and  $N_o$  is home-plate (point fixed in  $N$ ).



**MG road-map** for projectile motion  $x$  and  $y$  ( $\hat{n}_x, \hat{n}_y$  measures of  $Q$ 's position vector from  $N_o$ )

Variable	Translate/Rotate	Direction (unit vector)	System $S$	FBD of $S$	About point	MG road-map equation
$x$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<b>Draw</b>	Not applicable	<input type="text"/> · ( <input type="text"/> = <input type="text"/> (20.1))
$y$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<b>Draw</b>	Not applicable	<input type="text"/> · ( <input type="text"/> = <input type="text"/> (20.1))



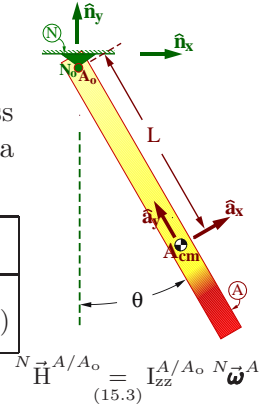
Draw FBD

Solution and simulation link at [www.MotionGenesis.com](http://www.MotionGenesis.com) ⇒ [Textbooks](#) ⇒ [Resources](#).

### 21.1.2 MG road-map: Rigid body pendulum (2D)

A non-uniform density rigid rod  $A$  is attached at point  $A_o$  of  $A$  by a frictionless revolute/pin joint to Earth  $N$  (Newtonian reference frame). The rod swings with a “pendulum angle”  $\theta$  in a vertical plane that is perpendicular to unit vector  $\hat{a}_z$ .

Variable	Translate/Rotate	Direction (unit vector)	System $S$	FBD of $S$	About point	MG road-map equation
$\theta$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<b>Draw</b>	<input type="text"/>	<input type="text"/> · ( <input type="text"/> = <input type="text"/> (20.4))

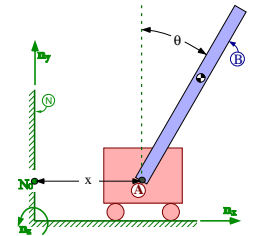


Solution and simulation link at [www.MotionGenesis.com](http://www.MotionGenesis.com) ⇒ [Textbooks](#) ⇒ [Resources](#).

$${}^N \vec{H}^{A/A_o} = I_{zz}^{A/A_o} {}^N \vec{\omega}^A \quad (15.3)$$

### 21.1.3 MG road-map: Inverted pendulum on cart ( $x$ and $\theta$ ) (2D)

A rigid rod  $B$  is pinned to a massive cart  $A$  (modeled as a particle) that translates horizontally in a Newtonian reference frame  $N$ . The cart's position vector from a point  $N_o$  fixed in  $N$  is  $x\hat{n}_x$  ( $\hat{n}_x$  is horizontally-right).  $B$ 's swinging motion in  $N$  is in a vertical plane perpendicular to  $\hat{n}_z$  (a unit vector fixed in both  $B$  and  $N$ ).



Variable	Translate/Rotate	Direction (unit vector)	System $S$	FBD of $S$	About point	MG road-map equation
$x$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<b>Draw</b>	Not applicable	<input type="text"/> · ( <input type="text"/> = <input type="text"/> (20.1))
$\theta$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<b>Draw</b>	<input type="text"/>	<input type="text"/> · ( <input type="text"/> = <input type="text"/> (20.4))

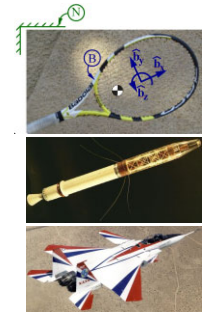
Homework 15.8 and Chapter 25 complete these calculations.

Note:  $m^S * {}^N \vec{a}^{S_{cm}} \stackrel{(11.3)}{=} m^A * {}^N \vec{a}^A + m^B * {}^N \vec{a}^{B_{cm}}$  and  $\frac{{}^N d}{dt} {}^N \vec{H}^{B/A} + \dots \stackrel{(20.6)}{=} I_{zz}^{B/A} * {}^N \vec{\alpha}^B + m^B * \vec{r}^{B_{cm}/A} \times {}^N \vec{a}^A$ .

### 21.1.4 MG road-map: Rotating rigid body (3D)

Shown right is a rotating rigid body  $B$  (e.g., tennis racquet, spacecraft, or aircraft) in a Newtonian reference frame  $N$ . Right-handed orthogonal unit vectors  $\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$  are fixed in  $B$ .

Variable	Translate/ Rotate	Direction (unit vector)	System $S$	FBD of $S$	About point	<i>MG road-map equation</i>
$\omega_x$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<b>Draw</b>	<input type="text"/>	<input type="text"/> $\cdot$ ( <input type="text"/> = <input type="text"/> (20.4))
$\omega_y$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<b>Draw</b>	<input type="text"/>	<input type="text"/> $\cdot$ ( <input type="text"/> = <input type="text"/> (20.4))
$\omega_z$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<b>Draw</b>	<input type="text"/>	<input type="text"/> $\cdot$ ( <input type="text"/> = <input type="text"/> (20.4))

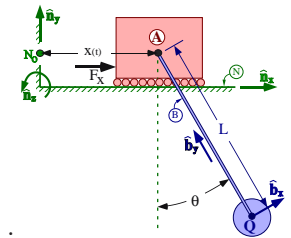


Solution and simulation link at [www.MotionGenesis.com](http://www.MotionGenesis.com)  $\Rightarrow$  [Textbooks](#)  $\Rightarrow$  [Resources](#).

Note: The “about point” is somewhat arbitrary. When  $B_{cm}$  is chosen:  ${}^N \vec{H}^{B/B_{cm}} = \overset{\equiv}{\mathbf{I}}^{B/B_{cm}} \cdot {}^N \vec{\omega}^B$ . (15.2)

### 21.1.5 MG road-map: Bridge crane equations of motion (2D)

A payload (particle)  $Q$  is welded to a light rigid cable  $B$  which swings in a Newtonian frame  $N$ . Cable  $B$  is pinned to a massive trolley  $A$  that can move horizontally along a smooth slot fixed in  $N$  with a **specified** (known) displacement  $x(t)$ . A translational actuator with force measure  $F_x$  connects trolley  $A$  to point  $N_o$  of  $N$ .



*MG road-map* for pendulum angle  $\theta$ , actuator force  $F_x$ , and cable tension

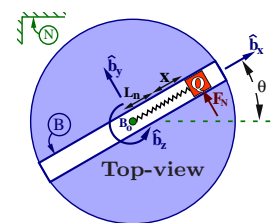
Variable	Translate/ Rotate	Direction (unit vector)	System $S$	FBD of $S$	About point	<i>MG road-map equation</i>
$\theta$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<b>Draw</b>	<input type="text"/>	<input type="text"/>
$F_x$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<b>Draw</b>	Not applicable	<input type="text"/>
Tension	<input type="text"/>	<input type="text"/>	<input type="text"/>	<b>Draw</b>	Not applicable	<input type="text"/>

Student/Instructor version at [www.MotionGenesis.com](http://www.MotionGenesis.com)  $\Rightarrow$  [Textbooks](#)  $\Rightarrow$  [Resources](#)

Note: Only the  $\theta$  road-map equation is needed to predict this system’s motion. The others are shown for illustrative purposes.

### 21.1.6 MG road-map: Particle on spinning slot (2D)

A particle  $Q$  slides on a straight slot  $B$ . The slot is connected with a revolute joint to a Newtonian frame  $N$  at point  $B_o$  so that  $B$  rotates in a horizontal plane perpendicular to  $\hat{\mathbf{b}}_z$  ( $\hat{\mathbf{b}}_z$  is vertically-upward and fixed in both  $B$  and  $N$ ).



Note: Homework 14.7 completes the MG road-map calculations for  $x$  and  $\theta$ .

*MG road-map* for  $x$ ,  $\theta$ , and  $F_N$  ( $\hat{\mathbf{b}}_y$  measure of normal force on  $Q$  from  $B$ )

Variable	Translate/ Rotate	Direction (unit vector)	System $S$	FBD of $S$	About point	<i>MG road-map equation</i>
$x$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<b>Draw</b>	Not applicable	<input type="text"/> $\cdot$ ( <input type="text"/> = <input type="text"/> )
$\theta$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<b>Draw</b>	$B_o$	<input type="text"/> $\cdot$ ( <input type="text"/> = <input type="text"/> )
$F_N$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<b>Draw</b>	Not applicable	<input type="text"/> $\cdot$ ( <input type="text"/> = <input type="text"/> )

Note: The  $F_N$  road-map equation is needed to predict motion if a friction force depends on  $\mu F_N$ .

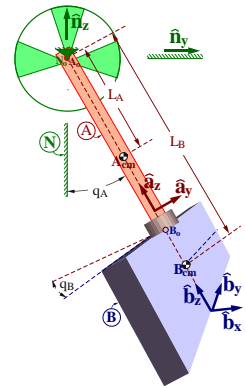
$${}^N \vec{H}^{S/B_o} = {}^N \vec{H}^{B/B_o} + {}^N \vec{H}^{Q/B_o} \quad \text{where} \quad {}^N \vec{H}^{B/B_o} = \overset{\equiv}{\mathbf{I}}_{zz}^{B/B_o} {}^N \vec{\omega}^B \quad \text{and} \quad {}^N \vec{H}^{Q/B_o} = \vec{\mathbf{r}}^{Q/B_o} \times m^Q {}^N \vec{\mathbf{v}}^Q. \quad (10.3)$$

### 21.1.7 MG road-map: Motion of a chaotic double pendulum (3D)

Shown right is a mechanical model of a swinging babyboot (uniform plate  $B$ ) attached by a shoelace (thin uniform rod  $A$ ) to a fixed rigid support  $N$ . Rod  $A$  is attached to  $N$  by a revolute joint at point  $N_o$  of  $N$ .  $B$  is attached to  $A$  with a second revolute joint at point  $B_o$  so  $B$  can rotate freely about  $A$ 's axis.

Note: The revolute joints' axes are *perpendicular*, not parallel.

- The plate, rod, and support are rigid.
- The revolute joints are ideal (massless, frictionless, no slop/flexibility).
- Earth is a Newtonian reference frame  $N$ .
- Forces due to Earth's gravitation are uniform and constant.
- Other distance forces (electromagnetic and gravitational) and air-resistance are negligible.



Right-handed sets of unit vectors  $\hat{n}_x, \hat{n}_y, \hat{n}_z$ ;  $\hat{a}_x, \hat{a}_y, \hat{a}_z$ ;  $\hat{b}_x, \hat{b}_y, \hat{b}_z$  are fixed in  $N, A, B$ , respectively, with  $\hat{n}_x = \hat{a}_x$  parallel to the revolute axis joining  $A$  to  $N$ ,  $\hat{n}_z$  vertically-upward,  $\hat{a}_z = \hat{b}_z$  parallel to the rod's long axis (and the revolute axis joining  $B$  to  $A$ ), and  $\hat{b}_z$  perpendicular to plate  $B$ .  $q_A$  is the angle from  $\hat{n}_z$  to  $\hat{a}_z$  with  $+\hat{n}_x$  sense.  $q_B$  is the angle from  $\hat{a}_y$  to  $\hat{b}_y$  with  $+\hat{a}_z$  sense.

Variable	Translate/ Rotate	Direction (unit vector)	System $S$	FBD of $S$	About point	<i>MG road-map equation</i>
$q_A$	<input type="text"/>	<input type="text"/>	<input type="text"/>	Draw	<input type="text"/>	<input type="text"/> · ( <input type="text"/> = <input type="text"/> )
$q_B$	<input type="text"/>	<input type="text"/>	<input type="text"/>	Draw	<input type="text"/>	<input type="text"/> · ( <input type="text"/> = <input type="text"/> )

Solution and simulation link at [www.MotionGenesis.com](http://www.MotionGenesis.com) ⇒ [Textbooks](#) ⇒ [Resources](#).

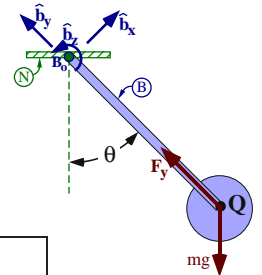
Note: The “about point” for the  $q_B$  road-map can be shifted from  $B_o$  to  $B_{cm}$  since  $\hat{b}_z \cdot \vec{M}^{B/B_{cm}} = \hat{b}_z \cdot \vec{M}^{B/B_o}$ . (17.4)

$${}^N \vec{H}^{S/A_o} = {}^N \vec{H}^{A/A_o} + {}^N \vec{H}^{B/A_o} \quad \text{where} \quad {}^N \vec{H}^{A/A_o} = I_{zz}^{A/A_o} {}^N \vec{\omega}^A \quad \text{and} \quad {}^N \vec{H}^{B/A_o} = \vec{I}^{B/B_{cm}} \cdot {}^N \vec{\omega}^B + \vec{r}^{B_{cm}/A_o} \times m^B {}^N \vec{v}^{B_{cm}}. \quad (15.3) \quad (15.4, 15.2)$$

### 21.1.8 MG road-map: Particle pendulum (2D) – angle and tension

A particle  $Q$  is welded to the distal end of a light rigid rope  $B$ . The rope's other end attaches to a point  $B_o$ , fixed in a Newtonian reference frame  $N$ . The swinging motion of  $B$  and  $Q$  is in a vertical plane that is perpendicular to unit vector  $\hat{b}_z$ .

*MG road-map* for pendulum angle  $\theta$  and tension  $F_y$  ( $\hat{b}_y$  measure of force on  $Q$  from  $B$ )



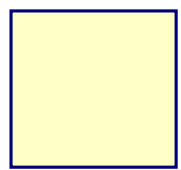
Variable	Translate/ Rotate	Direction (unit vector)	System $S$	FBD of $S$	About point	<i>MG road-map equation</i>
$\theta$	<input type="text"/>	<input type="text"/>	<input type="text"/>	Draw	<input type="text"/>	<input type="text"/> · ( <input type="text"/> = <input type="text"/> ) (20.4)
$F_y$	<input type="text"/>	<input type="text"/>	<input type="text"/>	Draw	Not applicable	<input type="text"/> · ( <input type="text"/> = <input type="text"/> ) (20.1)

Solution and simulation link at [www.MotionGenesis.com](http://www.MotionGenesis.com) ⇒ [Textbooks](#) ⇒ [Resources](#).

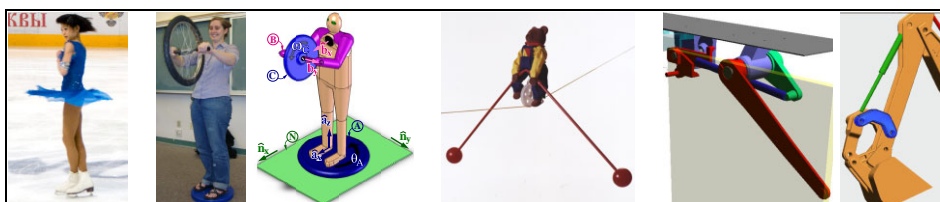
Note: Only the  $\theta$  road-map equation is needed to predict motion. The other is shown for illustrative purposes.

$$\text{Note: } {}^N \vec{H}^{S/N_o} = {}^N \vec{H}^{Q/N_o} = \vec{r}^{Q/N_o} \times m^Q {}^N \vec{v}^Q. \quad (10.3)$$

Section 24.3.2 completes all MG road-map calculations for  $\theta$ .



Draw FBDs



Many additional MG road-map examples at [www.MotionGenesis.com](http://www.MotionGenesis.com) ⇒ [Textbooks](#) ⇒ [Resources](#).

### 21.1.9 MG road-map: Dynamicist on a turntable (ice-skater)

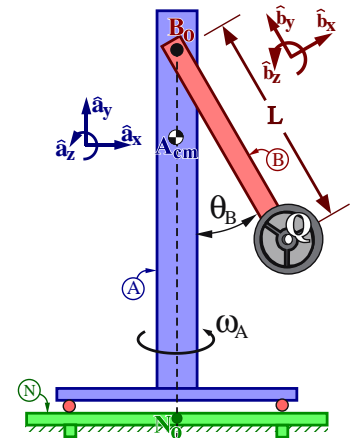
A dynamics instructor stands on a spinning turntable and swings a heavy dumbbell  $Q$  inward and outward to change his spin-rate (similar to the ice-skater).  $Q$  is modeled as a particle rigidly attached (welded) to the end of the instructor's hands.

The schematic (below-right) shows a rigid body  $A$  (modeling the instructor's legs, torso, and head) that rotates (without friction) relative to Earth (a Newtonian reference frame  $N$ ) about a vertical axis that is fixed in both  $A$  and  $N$  and which passes through both point  $N_o$  of  $N$  and point  $A_{cm}$  ( $A$ 's center of mass).

A massless rigid arm  $B$  (modeling the instructor's arms and hands) attaches to  $A$  by a revolute motor (shoulder/muscles) whose revolute axis is horizontal and located at point  $B_o$  of  $B$  ( $B_o$  lies on the vertical axis connecting  $N_o$  and  $A_{cm}$ ).

The motor (muscles) **specifies**  $B$ 's angle  $\theta_B$  relative to  $A$  to change in a known (prescribed) manner from 0 to  $\pi$  rad in 4 seconds ( $\theta_B = \pi \frac{t}{4}$ ).

Right-handed orthogonal unit vectors  $\hat{a}_x, \hat{a}_y, \hat{a}_z$  and  $\hat{b}_x, \hat{b}_y, \hat{b}_z$  are fixed in  $A$  and  $B$ , respectively, with  $\hat{a}_y$  vertically-upward,  $\hat{b}_z = \hat{a}_z$  parallel to the revolute motor's axis, and  $\hat{b}_y$  directed from  $Q$  to  $B_o$ .



Quantity	Symbol	Type	Value
Earth's gravitational constant	$g$	Constant	$9.8 \frac{m}{s^2}$
Distance between $Q$ and $B_o$	$L$	Constant	0.7 m
Mass of $Q$	$m$	Constant	12 kg
$A$ 's moment of inertia about line $\overline{A_{cm} B_o}$	$I_{yy}$	Constant	$0.6 \text{ kg m}^2$
Angle from $\hat{a}_y$ to $\hat{b}_y$ with $+\hat{a}_z$ sense	$\theta_B$	<b>Specified</b>	<b><math>0.25 \pi t</math> rad</b>
$\hat{a}_y$ measure of $A$ 's angular velocity in $N$	$\omega_A$	Variable	

Complete the **MG road-map** for the turntable's "spin-rate"  $\omega_A$  (Note: The "about point" is not unique)

Variable	Translate/ Rotate	Direction (unit vector)	System $S$	FBD of $S$	About point	<b>MG road-map equation</b>
$\omega_A$				<b>Draw</b>		

Student/Instructor version at [www.MotionGenesis.com](http://www.MotionGenesis.com)  $\Rightarrow$  [Textbooks](#)  $\Rightarrow$  [Resources](#)

### 21.1.10 MG road-map: Instructor on turntable with spinning wheel (3D)

The pictures to the right shows dynamicist Dr. G standing on a spinning turntable and holding a spinning bicycle wheel.



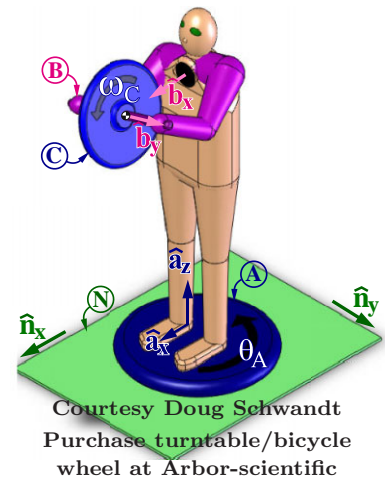
The mechanical model (below right) has a rigid body  $A$  (modeling the turntable, legs, torso, and head) that can freely rotate relative to Earth (Newtonian reference frame  $N$ ) about a vertical axis that is fixed in both  $A$  and  $N$  and which passes through the center of the turntable (point  $N_o$ ) and  $A_{cm}$  ( $A$ 's center of mass).

A light (massless) rigid frame  $B$  (modeling the shoulders, arms, hands, and a portion of the bicycle wheel's axle) is attached to  $A$  by a revolute motor at point  $B_o$  of  $B$  ( $B_o$  lies on the vertical axis passing through  $A_{cm}$ ). The motor's revolute axis passes through points  $B_o$  and  $C_{cm}$ , is horizontal, and is parallel to  $\hat{\mathbf{b}}_x = \hat{\mathbf{a}}_x$ .

A rigid bicycle wheel  $C$  is attached to  $B$  by a frictionless revolute joint whose axis passes through  $C_{cm}$  ( $C$ 's center of mass) and is parallel to  $\hat{\mathbf{b}}_y$ .

Right-handed orthogonal unit vectors  $\hat{\mathbf{a}}_x, \hat{\mathbf{a}}_y, \hat{\mathbf{a}}_z$  and  $\hat{\mathbf{n}}_x, \hat{\mathbf{n}}_y, \hat{\mathbf{n}}_z$  are fixed in  $A$  and  $N$ , respectively. Initially  $\hat{\mathbf{a}}_i = \hat{\mathbf{n}}_i$  ( $i = x, y, z$ ), and then rigid body  $A$  is subjected to a right-handed rotation characterized by  $\theta_A \hat{\mathbf{a}}_z$  where  $\hat{\mathbf{a}}_z = \hat{\mathbf{n}}_z$  is directed vertically-upward and  $\hat{\mathbf{a}}_x$  points from Dr. G's back to front (parallel to the axis of the revolute motor connecting  $A$  and  $B$ ).

Unit vectors  $\hat{\mathbf{b}}_x, \hat{\mathbf{b}}_y, \hat{\mathbf{b}}_z$  are fixed in  $B$ . Initially  $\hat{\mathbf{b}}_i = \hat{\mathbf{a}}_i$  ( $i = x, y, z$ ), then  $B$  is subjected to a  $\theta_B$  ( $\hat{\mathbf{a}}_x = \hat{\mathbf{b}}_x$ ) right-handed rotation in  $A$  where  $\hat{\mathbf{b}}_y$  is directed along the wheel's axle from Dr. G's right-to-left hand. Dr. G changes  $\theta_B$  in a **specified** sinusoid manner with amplitude  $30^\circ$  and period 4 seconds.

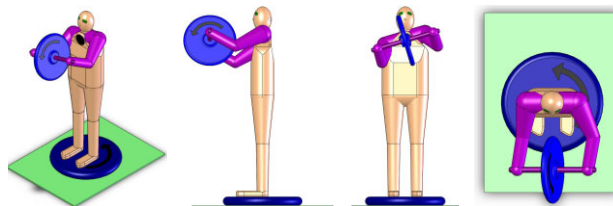


Quantity	Symbol and type	Value
Mass of $C$	$m^C$ Constant	2 kg
Distance between $B_o$ and $C_{cm}$	$L_x$ Constant	0.5 m
$A$ 's moment of inertia about $B_o$ for $\hat{\mathbf{a}}_z$	$I_{zz}^A$ Constant	0.64 kg m <sup>2</sup>
$C$ 's moment of inertia about $C_{cm}$ for $\hat{\mathbf{b}}_x$	$I^C$ Constant	0.12 kg m <sup>2</sup>
$C$ 's moment of inertia about $C_{cm}$ for $\hat{\mathbf{b}}_y$	$J^C$ Constant	0.24 kg m <sup>2</sup>
Angle from $\hat{\mathbf{n}}_x$ to $\hat{\mathbf{a}}_x$ with $+\hat{\mathbf{n}}_z$ sense	$\theta_A$ Variable	
Angle from $\hat{\mathbf{a}}_y$ to $\hat{\mathbf{b}}_y$ with $+\hat{\mathbf{a}}_x$ sense	$\theta_B$ <b>Specified</b>	$\frac{\pi}{6} \sin(\frac{\pi}{2} t)$
$\hat{\mathbf{b}}_y$ measure of $C$ 's angular velocity in $B$	$\omega_C$ Variable	

Complete the **MG road-map** for  $\theta_A$  and  $\omega_C$  (the "about points" are not unique).

Variable	Translate/Rotate	Direction (unit vector)	System $S$	FBD of $S$	About point	MG road-map equation
$\theta_A$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<b>Draw</b>	<input type="text"/>	<input type="text"/>
$\omega_C$	<input type="text"/>	<input type="text"/>	<input type="text"/>	<b>Draw</b>	<input type="text"/>	<input type="text"/>

Student/Instructor version at [www.MotionGenesis.com](http://www.MotionGenesis.com) ⇒ [Textbooks](#) ⇒ [Resources](#)



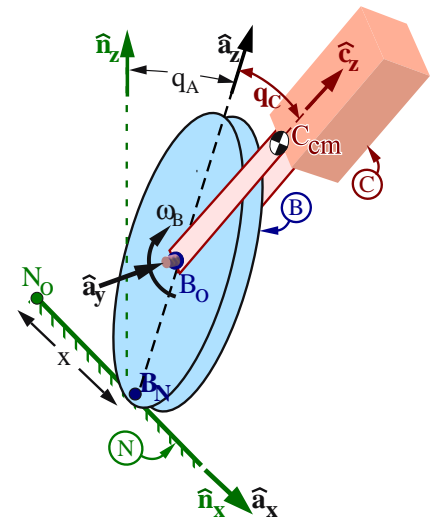
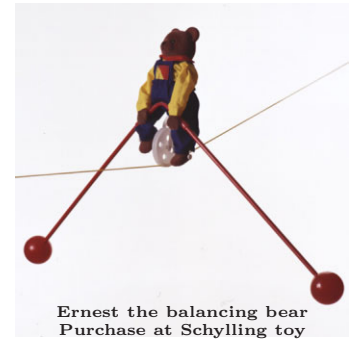
### 21.1.11 MG road-map: Bear riding a unicycle on a high-wire (3D)

The figures to the right show a (massless) pulley-wheel  $B$  that rolls along a taut (rigid) cable  $N$  (fixed on Earth, a Newtonian frame). Rigid body  $C$  (seat, rider, and balancing poles) attach to  $B$  with an ideal revolute motor at  $B_o$  ( $B$ 's centroid). The motor axis is aligned with  $B$ 's symmetry axis.

Right-handed orthogonal unit vectors  $\hat{n}_x, \hat{n}_y, \hat{n}_z$  are fixed in  $N$  with  $\hat{n}_z$  vertically-upward and  $\hat{n}_x$  directed horizontally along the cable from a point  $N_o$  (fixed in  $N$ ) to  $B_N$  ( $B$ 's rolling point of contact with  $N$ ).

Right-handed orthogonal unit vectors  $\hat{a}_x, \hat{a}_y, \hat{a}_z$  are directed with  $\hat{a}_x = \hat{n}_x$ ,  $\hat{a}_y$  parallel to the motor axis, and  $\hat{a}_z$  from  $B_N$  to  $B_o$ .

Right-handed unit vectors  $\hat{c}_x, \hat{c}_y, \hat{c}_z$  are parallel to  $C$ 's principal inertia axes about  $C_{cm}$  ( $C$ 's center of mass), with  $\hat{c}_y = \hat{a}_y$  and  $\hat{c}_z$  from  $B_o$  to  $C_{cm}$  (with balancing poles,  $C_{cm}$  is below  $B_o$  and  $L_C$  is negative).



Quantity	Symbol	Type	Value
Earth's gravitational constant	$g$	Constant	9.8 m/s <sup>2</sup>
Radius of $B$	$r_B$	Constant	30 cm
$\hat{c}_z$ measure of $C_{cm}$ 's position vector from $B_o$	$L_C$	Constant	-35 cm
Mass of $C$	$m^C$	Constant	2 kg
$C$ 's moment of inertia about $C_{cm}$ for $\hat{c}_x$	$I$	Constant	3.4 kg m <sup>2</sup>
$C$ 's moment of inertia about $C_{cm}$ for $\hat{c}_y$	$J$	Constant	3.2 kg m <sup>2</sup>
$C$ 's moment of inertia about $C_{cm}$ for $\hat{c}_z$	$K$	Constant	2.8 kg m <sup>2</sup>
$\hat{a}_y$ measure of motor torque on $B$ from $C$	$T_y$	<b>Specified</b>	below
Angle from $\hat{n}_z$ to $\hat{a}_z$ with $-\hat{n}_x$ sense	$q_A$	Variable	
$\hat{a}_y$ measure of ${}^A\omega^B$ ( ${}^A\omega^B = \omega_B \hat{a}_y$ )	$\omega_B$	Variable	
Angle from $\hat{a}_z$ to $\hat{c}_z$ with $+\hat{a}_y$ sense	$q_C$	Variable	
$\hat{n}_x$ measure of $\mathbf{r}^{B_N/N_o}$	$x$	Variable	

Form a complete set of **MG road-maps** for this systems's equations of motion (solution is not unique).  
If necessary, add more **MG road-maps** so there are the same number of equations as unknowns.

Variable	Translate/ Rotate	Direction (unit vector)	System $S$	FBD of $S$	About point	<b>MG road-map equation</b>	Additional Unknowns
$q_A$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<b>Draw</b>	<input type="checkbox"/>	<input type="text"/>	
$\omega_B$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<b>Draw</b>	<input type="checkbox"/>	<input type="text"/>	
$q_C$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<b>Draw</b>	<input type="checkbox"/>	<input type="text"/>	
$x$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<b>Draw</b>	<input type="checkbox"/>	<input type="text"/>	
* Additional scalar constraint equation(s): <input type="text"/>							

MG road-map for  $\omega_B$  is not unique. .

To move the unicycle to  $x_{Desired} = 10$  m, use a "PD control law" with  $T_y = -0.3(x - x_{Desired}) - 0.6\dot{x}$ .

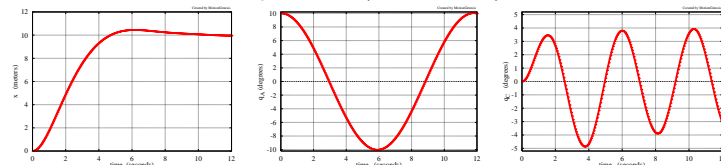
**Optional simulation:**

Plot  $x, q_A, q_C$  for  $0 \leq t \leq 12$  sec.

Use initial values:

$$x = 0 \text{ m} \quad q_A = 10^\circ \quad q_C = 0^\circ$$

$$\dot{x} = 0 \quad \dot{q}_A = 0 \quad \dot{q}_C = 0$$



Solution at [www.MotionGenesis.com](http://www.MotionGenesis.com) ⇒ [Get Started](#) ⇒ [Bear on rolling unicycle.](#)

### 21.1.12 MG road-map: Four-bar linkage statics (2D)

The figure to the right shows a planar four-bar linkage consisting of frictionless-pin-connected uniform rigid links  $A$ ,  $B$ , and  $C$  and ground  $N$ .

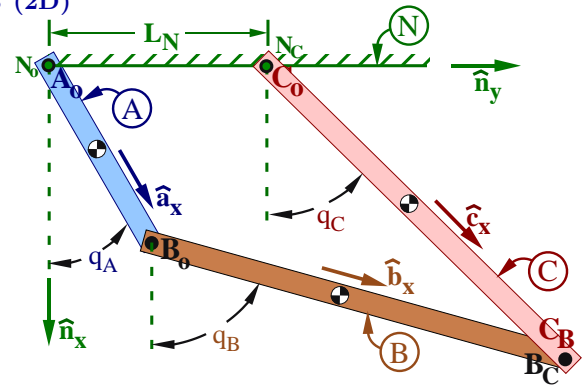
- Link  $A$  connects to  $N$  and  $B$  at points  $A_o$  and  $A_B$
- Link  $B$  connects to  $A$  and  $C$  at points  $B_o$  and  $B_C$
- Link  $C$  connects to  $N$  and  $B$  at points  $C_o$  and  $C_B$
- Point  $N_o$  of  $N$  is coincident with  $A_o$
- Point  $N_C$  of  $N$  is coincident with  $C_o$

Right-handed orthogonal unit vectors  $\hat{\mathbf{a}}_i$ ,  $\hat{\mathbf{b}}_i$ ,  $\hat{\mathbf{c}}_i$ ,  $\hat{\mathbf{n}}_i$  ( $i = x, y, z$ ) are fixed in  $A$ ,  $B$ ,  $C$ ,  $N$ , with:

- $\hat{\mathbf{a}}_x$  directed from  $A_o$  to  $A_B$
- $\hat{\mathbf{b}}_x$  directed from  $B_o$  to  $B_C$
- $\hat{\mathbf{c}}_x$  directed from  $C_o$  to  $C_B$
- $\hat{\mathbf{n}}_x$  vertically-downward
- $\hat{\mathbf{n}}_y$  directed from  $N_o$  to  $N_C$
- $\hat{\mathbf{a}}_z = \hat{\mathbf{b}}_z = \hat{\mathbf{c}}_z = \hat{\mathbf{n}}_z$  parallel to pin axes

As in Hw 8.7, create the following “**loop equation**” and dot-product with  $\hat{\mathbf{n}}_x$  and  $\hat{\mathbf{n}}_y$ .

$$L_A \hat{\mathbf{a}}_x + L_B \hat{\mathbf{b}}_x - L_C \hat{\mathbf{c}}_x - L_N \hat{\mathbf{n}}_y = \vec{\mathbf{0}}$$



Quantity	Symbol	Value
Length of link $A$	$L_A$	1 m
Length of link $B$	$L_B$	2 m
Length of link $C$	$L_C$	2 m
Distance between $N_o$ and $N_C$	$L_N$	1 m
Mass of $A$	$m^A$	10 kg
Mass of $B$	$m^B$	20 kg
Mass of $C$	$m^C$	20 kg
Earth's gravitational acceleration	$g$	$9.81 \frac{m}{s^2}$
$\hat{\mathbf{n}}_y$ measure of force applied to $C_B$	$H$	200 N
Angle from $\hat{\mathbf{n}}_x$ to $\hat{\mathbf{a}}_x$ with $+\hat{\mathbf{n}}_z$ sense	$q_A$	Variable
Angle from $\hat{\mathbf{n}}_x$ to $\hat{\mathbf{b}}_x$ with $+\hat{\mathbf{n}}_z$ sense	$q_B$	Variable
Angle from $\hat{\mathbf{n}}_x$ to $\hat{\mathbf{c}}_x$ with $+\hat{\mathbf{n}}_z$ sense	$q_C$	Variable

Complete the following **MG road-map** to determine this systems's **static configuration**.

Variable	Translate/ Rotate	Direction (unit vector)	System $S$	FBD of $S$	About point	<b>MG road-map equation</b>	Additional Unknowns
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<b>Draw</b>	<input type="checkbox"/>	<input type="checkbox"/>	$F_x^C, F_y^C$
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<b>Draw</b>	<input type="checkbox"/>	<input type="checkbox"/>	$F_x^C, F_y^C$
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<b>Draw</b>	<input type="checkbox"/>	<input type="checkbox"/>	$F_x^C, F_y^C$
* Additional scalar constraint equation:				$-L_A \sin(q_A) \dot{q}_A - L_B \sin(q_B) \dot{q}_B + L_C \sin(q_C) \dot{q}_C = 0$			
* Additional scalar constraint equation:				$L_A \cos(q_A) \dot{q}_A + L_B \cos(q_B) \dot{q}_B - L_C \cos(q_C) \dot{q}_C = 0$			

Determine the **static equilibrium** values of  $q_A$ ,  $q_B$ ,  $q_C$ .  
Use your intuition (guess), circle the **stable** solution.

Solution 1	$q_A \approx 20.0^\circ$	$q_B \approx 71.7^\circ$	$q_C = 38.3^\circ$
Solution 2	$q_A \approx 249.3^\circ$	$q_B \approx 140.2^\circ$	$q_C = 199.1^\circ$
Solution 3	$q_A \approx 30.7^\circ$	$q_B \approx 226.1^\circ$	$q_C = 254.7^\circ$

Solution at [www.MotionGenesis.com](http://www.MotionGenesis.com)  $\Rightarrow$  [Get Started](#)  $\Rightarrow$  Four-bar linkage

