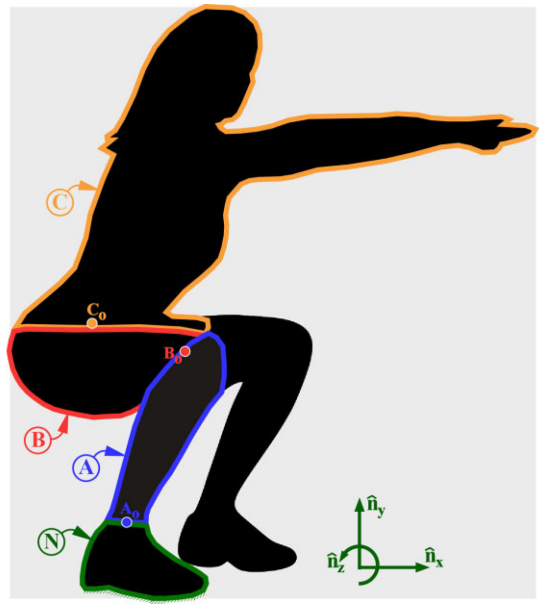
Chapter 23

Dynamics with MG road-maps

Motivating example – inverted triple pendulum (examples in Hw 18, 19, 21, 22)

The figure to the right shows a woman who is in the process of standing upright (transition from sitting to standing is difficult for the elderly, whereas athletes train with squats). Knowing the woman's motion (e.g., from motion capture data), dynamics can be used to determine the joint torques needed for a transition to upright standing. These joint torques are useful for biomechanics and robotics (e.g., athletic training, physical therapy, surgical strategies, and sizing motors for robotic assist).



A one-sided simplified 2D model can be used to analyze this maneuver. The model has the foot fixed to ground (Newtonian reference frame N). Rigid lower-leg A is connected to the foot via an ideal revolute motor at point A_o of A that exerts a torque T_A on A. Rigid upper-leg B is connected to A via a revolute motor at point B_o of B that exerts a torque T_B on B. The rigid head-arms-torso C is connected to B via a revolute motor at point C_o of C that exerts a torque T_C on C. The unit vector $\widehat{\mathbf{n}}_z$ is parallel to the axis of each revolute motor.

An inefficient way to form equations for T_A, T_B, T_C is to use the Newton/Euler equations and ***free-body diagrams (FBDs)*** for each of A, B, C. For a 3D analysis, this "brute force" technique requires **15** equations and **15** unknowns, and even a 2D analysis, requires **9** equations and **9** unknowns!

The methods of D'Alembert, Lagrange, Kane form only **3** equations for **3** unknowns (T_A, T_B, T_C), avoiding extra equations and unnecessary unknowns. For this example, D'Alembert's method cleverly chooses directions, systems, and variations of Newton/Euler equations [equation (22.4)] to form only **3** equations.

Clever efficient dynamics $\Rightarrow$

$$\widehat{\mathbf{n}}_z \cdot \left(\vec{\mathbf{M}}^{S_1/C_o} = \frac{{}^N d \; {}^N\vec{\mathbf{H}}^{S_1/C_o}}{dt} + {}^N\vec{\mathbf{v}}^{C_o} \times {}^N\vec{\mathbf{L}}^{S_1} \right) \qquad S_1 \text{ is only } C.$$

$$\widehat{\mathbf{n}}_z \cdot \left(\vec{\mathbf{M}}^{S_2/B_o} = \frac{{}^N d \; {}^N\vec{\mathbf{H}}^{S_2/B_o}}{dt} + {}^N\vec{\mathbf{v}}^{B_o} \times {}^N\vec{\mathbf{L}}^{S_2} \right) \qquad S_2 \text{ is } B \text{ and } C.$$

$$\widehat{\mathbf{n}}_z \cdot \left(\vec{\mathbf{M}}^{S_3/A_o} = \frac{{}^N d \; {}^N\vec{\mathbf{H}}^{S_3/A_o}}{dt} \right) \qquad S_3 \text{ is } A,\, B,\, C.$$

D'Alembert's motivation for these equations is that they eliminate the irrelevant constraint forces between A, B, C – hence there are only **3** equations for the **3** unknowns (T_A, T_B, T_C). The next section demonstrates ***MG road-maps***, a new systematic way to understand/employ D'Alembert's clever method.

A modern way to efficiently form static or dynamic equations with FBDs is to:[1]

- Choose ***scalar variables*** that describe the relevant ***unknown*** configuration, motion, or forces.
- Complete the associated ***MG road-maps*** and ***free-body diagrams***.[2]
- Complete the calculations specified by the ***MG road-map equation***.

MG road-map for efficient statics and dynamics.

Variable	Translate/ Rotate	Direction (unit vector)	System S	FBD of S	About point	***MG road-map equation***	Additional Unknowns
				Draw			?

* If applicable: Additional ***constraint equations*** and their time-derivatives (e.g., closed linkages or **rolling**).

Column	Enter the following information
1	**Unknown scalar variable** (e.g., a position, velocity, force, or torque variable).
2	Type of motion associated with the variable: **translate** or **rotate**.
3	Vector direction (e.g., **unit vector** $\hat{\mathbf{u}}$) associated with the direction of motion.
4	List of objects whose motion (e.g., velocity or angular velocity) is directly effected by the variable in column 1 ("freeze" any variable other than the variable in column 1 and decide what objects **must** move). This picks a **system** S that reduces/eliminates constraint forces. Note: If the variable in column 1 is a force measure, treat it as a velocity measure and determine what objects move. If it is a torque measure, treat it as an angular velocity measure and determine what objects move.
5	**Draw** a ***free-body diagram*** of **system** S (**draw** relevant contact/distance forces). Note: See force/torque models for gravity, springs, dampers, air-resistance, etc., in Chapter 21.
6	If column 2 was **rotate**, choose a point O (or line L) about which moments are to be taken. Note: Choose point O to eliminate moments of unknown forces (e.g., contact forces on S) – look at FBD. Note: To facilitate calculations, you can slide the "about point" along the line L parallel to $\hat{\mathbf{u}}$. This is because $\hat{\mathbf{u}} \cdot \vec{\mathbf{M}}^{S/O} = \hat{\mathbf{u}} \cdot \vec{\mathbf{M}}^{S/P}$ if both points O and P are on line L (proved in Section 19.1.3).
7a	If column 2 was **translate**, use: (*N* is a Newtonian reference frame) $\boxed{\hat{\mathbf{u}} \cdot \left(\vec{\mathbf{F}}^S \underset{\text{Dynamics}}{=} m^S * {}^N\vec{\mathbf{a}}^{S_{\text{cm}}}\right)}$ or $\boxed{\hat{\mathbf{u}} \cdot \vec{\mathbf{F}}^S \underset{\text{Statics}}{=} 0}$
	To calculate $m^S * {}^N\vec{\mathbf{a}}^{S_{\text{cm}}}$ for a system S of objects A, B, C: $\boxed{m^S * {}^N\vec{\mathbf{a}}^{S_{\text{cm}}} \underset{(13.3)}{=} m^A * {}^N\vec{\mathbf{a}}^{A_{\text{cm}}} + m^B * {}^N\vec{\mathbf{a}}^{B_{\text{cm}}} + m^C * {}^N\vec{\mathbf{a}}^{C_{\text{cm}}}}$
7b	If column 2 was **rotate**, use: $\boxed{\hat{\mathbf{u}} \cdot \left(\vec{\mathbf{M}}^{S/O} \underset{(22.4)}{=} \frac{{}^N d\, {}^N\vec{\mathbf{H}}^{S/O}}{dt} + \dots\right)}$ or $\boxed{\hat{\mathbf{u}} \cdot \vec{\mathbf{M}}^{S/O} \underset{\text{Statics}}{=} 0}$
	For a system S with a particle Q and rigid body B: $\boxed{{}^N\vec{\mathbf{H}}^{S/O} = {}^N\vec{\mathbf{H}}^{Q/O} + {}^N\vec{\mathbf{H}}^{B/O}}$ For particle Q: ${}^N\vec{\mathbf{H}}^{Q/O} \underset{(12.3)}{=} \vec{\mathbf{r}}^{Q/O} \times m^Q\, {}^N\vec{\mathbf{v}}^Q$ For rigid body B: ${}^N\vec{\mathbf{H}}^{B/O} \underset{(17.2)}{=} {}^N\vec{\mathbf{H}}^{B/B_p} + \vec{\mathbf{r}}^{B_p/O} \times m^B\, {}^N\vec{\mathbf{v}}^{B_{\text{cm}}}$ where: ${}^N\vec{\mathbf{H}}^{B/B_p} \underset{(17.1)}{=} \overset{\Rightarrow}{\mathbf{I}}^{B/B_p} \cdot {}^N\vec{\boldsymbol{\omega}}^B + \vec{\mathbf{r}}^{B_{\text{cm}}/B_p} \times m^B\, {}^N\vec{\mathbf{v}}^{B_p}$
8*	Additional constraint force/torque variables may appear in MG road-maps. If applicable, append configuration or motion ***constraints*** (e.g., closed linkages or **rolling**) that interrelate Column 1 position/velocity variables.

See MG road-map examples in next sections and at **www.MotionGenesis.com** ⇒ **Textbooks** ⇒ **Resources**.

[1] Mitiguy and Fregly invented ***MG road-maps*** to efficiently form static and dynamic equations. ***MG road-maps*** combine the simplicity of a spreadsheet with the physical insights of free-body diagrams. ***MG road-maps*** were inspired by the cleverness of D'Alembert and the mathematical rigor of Kane and Euler/Lagrange. ***MG road-maps*** are easy to teach/learn and approximate the efficiency of Kane's method for most systems (except embedded constraints like rolling/gears).

[2] ***Free-body diagrams (FBDs)*** are a means to an end, not an end in itself. ***MG road-maps*** help determine **which FBDs** to draw and **what to do** with them, – which differs significantly from knowing **how** to draw FBDs.