

$\vec{M} = \frac{d\vec{H}}{dt}$: **FBD, MG road-maps, and concepts for rotating rigid bodies.**

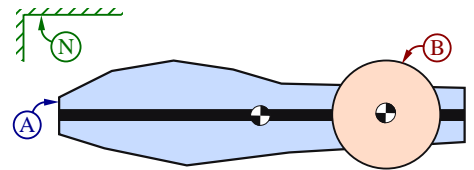
19.1 Concept: Useful equations for 3D mechanics?

- $\vec{F} = m\vec{a}$ is useful for analyzing 3D translational motions of a rigid body. **True/False**
- $\vec{M} = I\vec{\alpha}$ is useful for analyzing 3D rotational motions of a rigid body. **True/False**

19.2 Concept: Rotational motion? (Draw free-body diagrams.)

The figure to the right shows a rigid body B connected to a rigid body A with a **torque/revolute motor** (not shown).

Initially, A and B are **at rest** (stationary) in deep empty space in a Newtonian (inertial) reference frame N .



- It is possible for the torque motor to rotate A in N . **True/False**
- It is possible for the torque motor to rotate B in N . **True/False**
- †**Optional:** Suppose revolute motor's axis of rotation is initially parallel to a vector λ fixed in N . Can the motor cause this axis to point in another direction (i.e., so that at some later time the revolute axis is not parallel to λ)? **Yes/No**. (Hint: See Homework 15.13.)
Explain: B or A must have a non-zero in a direction associated with λ .

19.3 Concept: When are both $\vec{F}^S = \vec{0}$ and $\vec{M}^{S/P} = \vec{0}$ valid? (Draw free-body diagrams.)

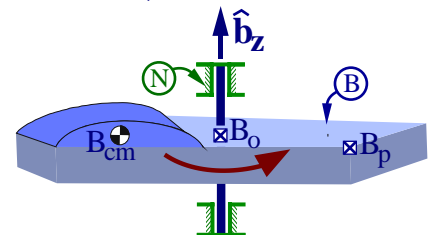
Consider a system S on Earth (a Newtonian reference frame) and the quantities \vec{F}^S (the resultant of all forces on S) and $\vec{M}^{S/P}$ (the moment of all forces on S about an arbitrary point P).

| For each situation below, determine whether both $\vec{F}^S = \vec{0}$ and $\vec{M}^{S/P} = \vec{0}$ (P is any point) apply. | |
|---|---------------|
| S is a baseball (particle) dropped from the roof of a two-story building (small air-resistance). | Yes/No |
| S is a baseball (particle) falling at terminal velocity. | Yes/No |
| S is a set of baseballs (particles) falling at terminal velocity. | Yes/No |
| S is a car (rigid body) parked on a hill. | Yes/No |
| S is a car (rigid body) whose every point moves on a straight line at constant speed. | Yes/No |
| S is a car moving on a straight line whose speed changes from 0 to 30 mph in 5 sec. | Yes/No |
| S is a car moving on a circular curve at constant speed. | Yes/No |
| S is a heavy (massive) spring between a wall and a vibrating object (S is just the spring). | Yes/No |
| S is a light (massless) spring between a wall and vibrating object (S is just the spring). | Yes/No |
| S is symmetric rigid windmill blades (with fixed center) spinning with constant angular speed. | Yes/No |
| S is symmetric rigid windmill blades (with fixed center) spinning with varying angular speed. | Yes/No |

... the whole burden of philosophy seems to consist in this, from the phenomena of motions to investigate the forces of nature, and from the forces to demonstrate the other phenomena. -Isaac Newton, *Principia Philosophiae* (1686)

19.4 Proof of Euler's special 2D rigid body formula (simple angular velocity).

Consider an oddly-shaped rigid body B supported by two rusty (friction-filled) bearings so B spins with a simple angular velocity ${}^N\vec{\omega}^B = \omega_z \hat{b}_z$ in a Newtonian reference frame N , where \hat{b}_z is **fixed** in both B and N , parallel to the bearings' axes, and directed vertically upward (opposite gravity). Point B_o of B is at the midpoint between the bearings (along B 's axis of rotation in N).



Denoting \vec{M}^{B/B_p} as the moment of all forces on B about B_p (a point **fixed** on B), and starting with