7.1 ♣ Notation, words, pictures for position, velocity, and acceleration. (Sections 3.1 and 8.1)

Complete each blank with a word: | point | reference frame | position | velocity | acceleration |

$\vec{\mathbf{r}}^{\,Q/P}$ is point Q's ______ vector from ______ P.	${}^{N}\vec{\mathbf{v}}^{\,Q}$ is point Q's ______ in ______ N.	${}^{N}\vec{\mathbf{a}}^{\,Q}$ is point Q's ______ in ______ N.
Draw P, Q, and $\vec{\mathbf{r}}^{\,Q/P}$.	**Draw** Q and N.	

7.2 ♣ What is a point and a particle? (Section 3.1)

To visualize center of mass, draw a doughnut.

Statement	True or False
A point has all the attributes of a particle.	**True/False**
A particle has all the attributes of a point.	**True/False**
A point with mass (massive point) is a particle.	**True/False**
The center of mass of a rigid body is a point.	**True/False**
The center of mass of a rigid body is a particle.	**True/False**

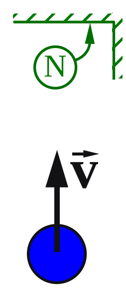
7.3 ♣ Concept: What objects have a unique velocity/acceleration? (Section 8.1)

The velocity $\vec{\mathbf{v}}$ of some object S relative to Earth is to be determined.
This object S could be a (circle ***all*** objects that have an ***unambiguously*** defined velocity $\vec{\mathbf{v}}$):

Real number	Line	Set of points	Center of a circle
Vector	Triangle	Reference frame	Mass center of set of particles
Matrix	Point	Rigid body	Mass center of a rigid body
3D orthogonal basis	Particle	Flexible body	System of particles and bodies

Repeat for the acceleration $\vec{\mathbf{a}}$ of some object S relative to Earth $\boxed{\text{box appropriate objects}}$.

7.4 ♣ Concept: Velocity, acceleration, and differentiation. (Sections 1.6.1 and 8.1)



A baseball (particle) is thrown straight upward on Earth (a Newtonian reference frame N). Knowing the baseball's velocity $\vec{\mathbf{v}} = \vec{\mathbf{0}}$ when the ball reaches maximum height and Earth's gravitational acceleration constant $g \approx 9.8 \frac{\text{m}}{\text{s}^2}$, decide if the following statement about $\vec{\mathbf{a}}$ (the ball's acceleration in N) is true. If false, box the incorrect part of the statement.

$$\vec{\mathbf{a}} \triangleq \frac{{}^{N}d\vec{\mathbf{v}}}{dt} = \frac{{}^{N}d\,(\vec{\mathbf{0}})}{dt} = \frac{d\,\vec{\mathbf{0}}}{dt} = \vec{\mathbf{0}} \qquad \textbf{True/False}$$

Explain:

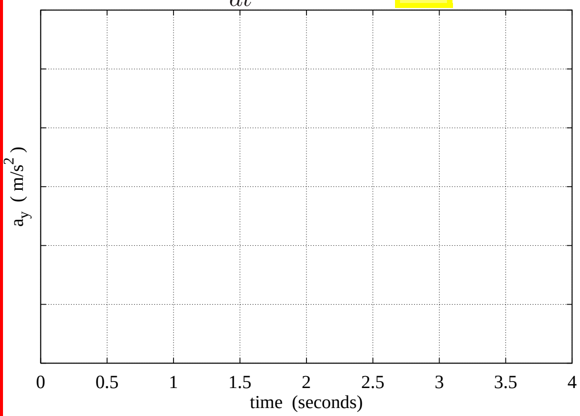
7.5 ♣ FE/EIT Review: Graphing $\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$ for a sky-diver and rocket-sled.

Complete the missing statements, axes values, and graphs. Use Earth's gravitational acceleration $g \approx 10\ \frac{\text{m}}{\text{s}^2}$.

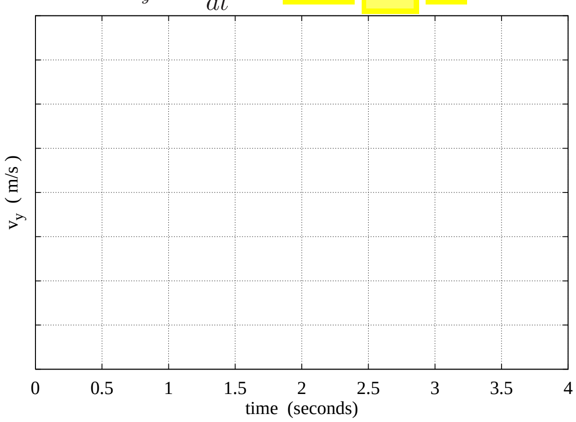
A sky-diver free-falls for 4 seconds after leaving a stationary helicopter from a height $y = 200$ m above Earth (y is positive-upward).

The only relevant force is Earth's gravity.

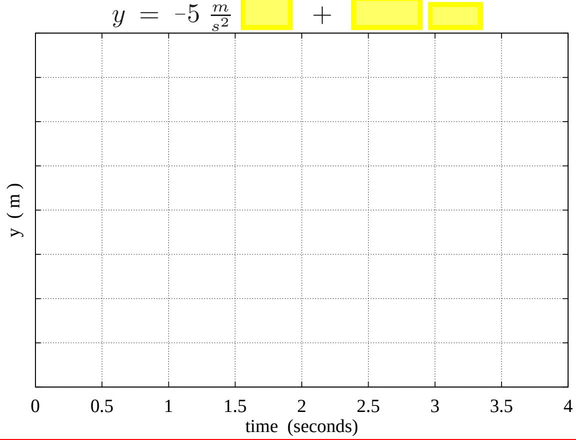
$$a_y \triangleq \frac{d^2y}{dt^2} = \square\ \square$$



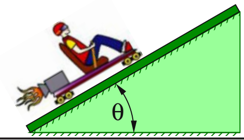
$$v_y \triangleq \frac{dy}{dt} = \square\ \square\ \square$$



$$y = -5\ \frac{m}{s^2}\ \square + \square\ \square$$

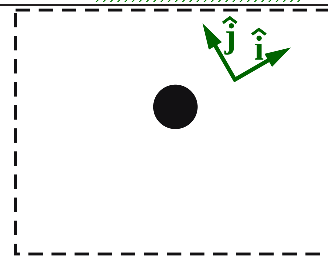


A rocket-sled of mass m is thrust along smooth inclined rails with time-varying force F_T.
The variable x measure's the sled's position along the rails.
Initially, $x = 0$ and $\dot{x} = 0$.



FBD. Draw forces

Below: Form $\vec{\mathbf{F}}_{\text{Net}}$ and then set $\vec{\mathbf{F}}_{\text{Net}} = m\,\vec{\mathbf{a}}$.
Use symbols m, g, F_T, F_N, θ.



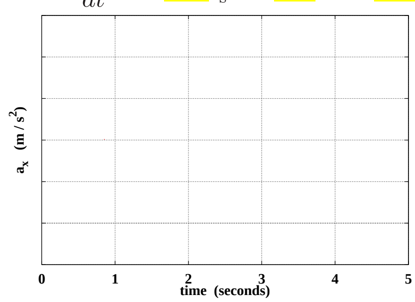
$$\square\ \hat{\mathbf{i}} + \square\ \hat{\mathbf{j}} = m\ \square\ \hat{\mathbf{i}}$$

$$\hat{\mathbf{i}}:\ \square = \square \quad\Rightarrow\quad \frac{d^2x}{dt^2} = \square - \square$$

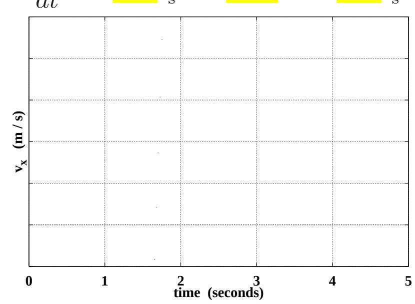
$$\hat{\mathbf{j}}:\ \square = \square \quad\Rightarrow\quad F_N = \square$$

Use $\theta = 30°$, $m = 100$ kg, $F_T = 600\ \frac{\text{N}}{\text{s}} * t$ for the following.

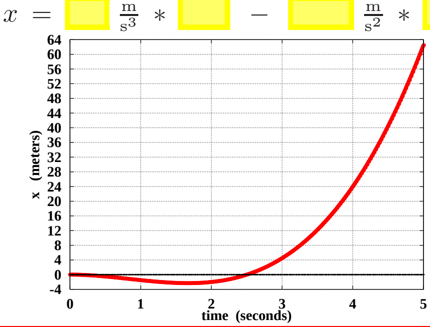
$$a_x \triangleq \frac{dv_x}{dt} = \square\ \frac{\text{m}}{\text{s}^3} * \square - \square\ \frac{\text{m}}{\text{s}^2}$$



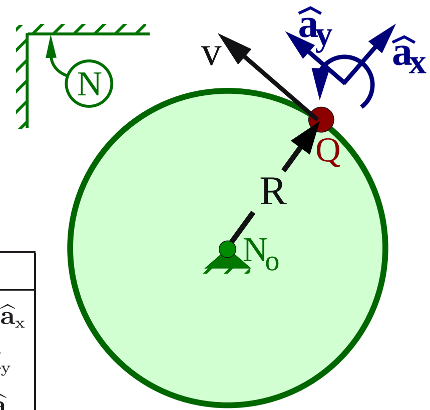
$$v_x \triangleq \frac{dx}{dt} = \square\ \frac{\text{m}}{\text{s}^3} * \square - \square\ \frac{\text{m}}{\text{s}^2} * \square$$



$$x = \square\ \frac{\text{m}}{\text{s}^3} * \square - \square\ \frac{\text{m}}{\text{s}^2} * \square$$



7.6 **Calculating centripetal acceleration in 2D.** **(Section 8.1)**
The following figure shows a point Q moving on a circle.
The circle is centered at N_o and fixed in a reference frame N.



A right-handed orthogonal unitary basis A is oriented with:

- $\hat{\mathbf{a}}_x$ directed from N_o to Q
- $\hat{\mathbf{a}}_z$ outward normal to the circle/paper.

Description	Symbol & type		Related
Radius of circle (distance between N_o and Q)	R	Constant	$\vec{\mathbf{r}}^{\,Q/N_o} = R\,\hat{\mathbf{a}}_x$
$\hat{\mathbf{a}}_y$ measure of Q's velocity in N	v	Variable	${}^{N}\vec{\mathbf{v}}^{Q} = v\,\hat{\mathbf{a}}_y$
$\hat{\mathbf{a}}_z$ measure of A's angular velocity in N	ω	Variable	${}^{N}\vec{\boldsymbol{\omega}}^{A} = \omega\,\hat{\mathbf{a}}_z$

(a) Write the definition of Q's velocity in N. **Show each step** to write v in terms of ω.
Result: (Also rearrange to solve for ω in terms of v.)

$$ {}^{N}\vec{\mathbf{v}}^{Q} \underset{(8.1)}{\triangleq} \boxed{} = v\,\hat{\mathbf{a}}_x \quad \underset{\text{Show work}}{\Rightarrow} \quad v = \boxed{} \quad \underset{\text{Rearrange}}{\Rightarrow} \quad \omega = \boxed{} $$

(b) Calculate Q's acceleration in N, first in terms of $R,\ \omega,\ \dot{\omega}$, then in terms of $R,\ v,\ \dot{v}$.
Result: (expressed in terms of $\hat{\mathbf{a}}_x,\ \hat{\mathbf{a}}_y,\ \hat{\mathbf{a}}_z$).

$$ {}^{N}\vec{\mathbf{a}}^{Q} = \underbrace{\boxed{}\,\hat{\mathbf{a}}_x + \boxed{}\,\hat{\mathbf{a}}_y}_{\text{In terms of } R,\ \omega,\ \dot{\omega}} = -\frac{\boxed{}}{\boxed{}}\,\hat{\mathbf{a}}_x + \boxed{}\,\hat{\mathbf{a}}_y $$

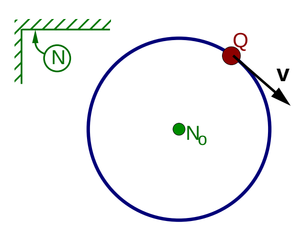
(c) Consider the situation when Q moves with ***constant*** speed on the circle (i.e., v is constant).
Express ${}^{N}\vec{\mathbf{a}}^{Q}$ in terms of $\vec{\boldsymbol{\omega}}^2$ ($\vec{\boldsymbol{\omega}}$ is A's angular velocity in N) and $\vec{\mathbf{r}}$ (Q's position vector from N_o).
Result: ${}^{N}\vec{\mathbf{a}}^{Q} = -\vec{\boldsymbol{\omega}}^2\,\vec{\mathbf{r}}$

Hint: Rearrange your previous result or use equation (8.3) and eliminate cross-products via the vector identity $\vec{\mathbf{a}} \times (\vec{\mathbf{b}} \times \vec{\mathbf{c}}) = \vec{\mathbf{b}}\,(\vec{\mathbf{a}} \cdot \vec{\mathbf{c}}) - \vec{\mathbf{c}}\,(\vec{\mathbf{a}} \cdot \vec{\mathbf{b}})$.

(d) The direction of Q's centripetal acceleration in N is (choose one):
From N_o to Q / From Q to N_o / Tangent to the circle / Outward normal to circle/ Other.

(e) As described in Section 8.5:
The literal meaning of the word "centripetal" is "**center-**$\boxed{}$".
The literal meaning of the word "centrifugal" is "**center-**$\boxed{}$".

7.7 ♣ **Velocity and acceleration concepts: What does "constant" mean?** **(Section 6.3 and Hw 7.6)**
The figure to the right shows a point Q moving with **constant speed** on a circle centered at N_o and fixed in a reference frame N. The following questions refer to $\vec{\mathbf{v}}$ (Q's velocity in N) and $\vec{\mathbf{a}}$ (Q's acceleration in N).



The magnitude of $\vec{\mathbf{a}}$ is constant	**True/False**
$\vec{\mathbf{a}}$ is constant in N	**True/False**
$\vec{\mathbf{v}}$ is constant in N	**True/False**