Complex numbers, circuits, Laplace transforms, frequency response, motors and sensors

Show work – except for ♣ fill-in-blanks (print .pdf from www.MotionGenesis.com ⇒ Textbooks ⇒ Resources).

8.1 Euler's formula and trigonometry functions (Chapter 15).

Show every step to express the right-hand side of the following expressions in terms of trigonometric functions of the real scalar variable θ (without the imaginary number i).

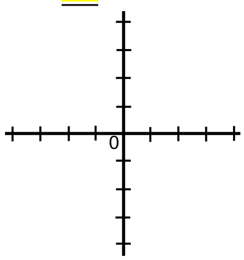
$$\frac{e^{i\,\theta} + e^{-i\,\theta}}{2} = \boxed{} \qquad\qquad \frac{e^{i\,\theta} - e^{-i\,\theta}}{2\,i} = \boxed{}$$

8.2 ♣ Putting real, imaginary, and complex numbers in magnitude-phase form (Section 15.1).

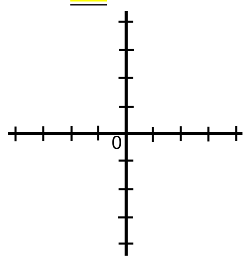
Clearly **mark** each of the following number's location in the complex plane.
Next, express the number in the magnitude-phase form $z = |z|\, e^{(\theta + 2\,\pi\,n)\,i}$, where

- $|z|$ is the magnitude of z and n is any integer (e.g., $n = 0, 1, 2, \ldots$)
- $-\pi \le \theta \le \pi$ is the angle between the positive real axis and the line connecting 0 to z

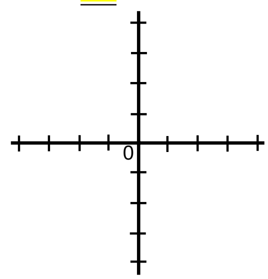
$2 = \underline{\boxed{}}\, e^{(\underline{\boxed{}} + 2\,\pi\,n)\,i}$



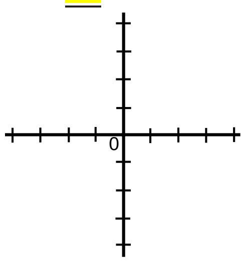
$-2 = \underline{\boxed{}}\, e^{(\underline{\boxed{}} + 2\,\pi\,n)\,i}$



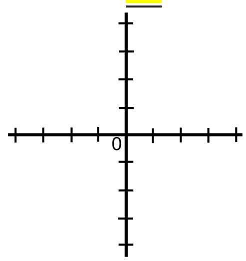
$4\,i = \underline{\boxed{}}\, e^{(\underline{\boxed{}} + 2\,\pi\,n)\,i}$



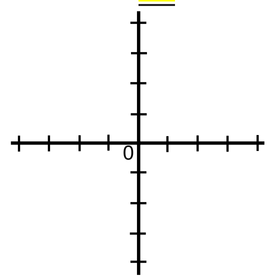
$-4\,i = \underline{\boxed{}}\, e^{(\underline{\boxed{}} + 2\,\pi\,n)\,i}$



$3.4641 + 2\,i = \underline{\boxed{}}\, e^{(\underline{\boxed{}} + 2\,\pi\,n)\,i}$



$-3.4641 - 2\,i = \underline{\boxed{}}\, e^{(\underline{\boxed{}} + 2\,\pi\,n)\,i}$



8.3 ♣ Why does multiplying two negative numbers produce a positive number? (Section 15.5)

Using magnitude-phase form, show $(-2) * (-2) = +4$.

$$(-2) * (-2) = \boxed{} * \boxed{} = 4\,e^{\boxed{}} = 4\,\cos(\boxed{} + \boxed{}) + \boxed{} = 4$$

8.4 ♣ Complex numbers and exponentiation (Section 15.6).

Find ***all*** complex numbers (in Cartesian form) equal to the following.

$\sqrt{4} = \boxed{} = \underline{\boxed{}}$ or $\underline{\boxed{}}$

$\sqrt{i} = \boxed{} \approx \underline{0.707} + \underline{0.707}\,i$ or $\underline{\boxed{}} - \underline{\boxed{}}\,i$

$1^{\frac{1}{2\pi}} = \boxed{} = 1,\ \ 0.54 + 0.84\,i,\ \ \boxed{}\ \ \ldots$