Show work – except for ♣ fill-in-blanks (print .pdf from www.MotionGenesis.com ⇒ Textbooks ⇒ Resources).

12.1 ♣ Matrix rows and columns

Given: $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ Row 1 of M $= [\, 1 \;\; 2 \,]$ Row 2 of M $= [\, 3 \;\; 4 \,]$

$M_{2,1} = 3$ Column 1 of M $= \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ Column 2 of M $= \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

12.2 ♣ Matrix transpose

$$\text{Transpose} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \qquad \text{Transpose} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

12.3 ♣ Matrix addition and subtraction (+ , -)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix} \qquad \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} - \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} = \begin{bmatrix} a-2 & b-4 & c-6 \\ d-3 & e-5 & f-7 \end{bmatrix}$$

12.4 ♣ Scalar-matrix multiplication (∗)

$$5 * \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix} \qquad 5 * \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 5a & 5b & 5c \\ 5d & 5e & 5f \end{bmatrix}$$

12.5 ♣ Matrix-matrix multiplication (∗)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3a + 5b \\ 3c + 5d \end{bmatrix} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 3 & x \\ 5 & y \end{bmatrix} = \begin{bmatrix} 3a + 5b & ax + by \\ 3c + 5d & cx + dy \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} * [\, x \;\; y \,] = \begin{bmatrix} ax & ay \\ bx & by \end{bmatrix} \qquad \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} * \begin{bmatrix} x & 3 \\ y & 5 \\ z & 7 \end{bmatrix} = \begin{bmatrix} ax+by+cz & 3a+5b+7c \\ dx+ey+fz & 3d+5e+7f \end{bmatrix}$$

12.6 ♣ Matrix determinants

$\det [\, 5 \,] \triangleq 5$

$\det [\, a \,] \triangleq a$

$$\det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = 1 * 4 - 2 * 3 = -2 \qquad \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ab - cd$$

Calculate the following determinant three ways: expand along the 1st row, 1st column, 2nd row.

$$\det \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 9 \end{bmatrix} = {}^{+}1 \det \begin{bmatrix} 5 & 6 \\ 0 & 9 \end{bmatrix} + {}^{-}2 \det \begin{bmatrix} 4 & 6 \\ 7 & 9 \end{bmatrix} + {}^{+}3 \det \begin{bmatrix} 4 & 5 \\ 7 & 0 \end{bmatrix} = -48$$

$$= {}^{+}1 \det \begin{bmatrix} 5 & 6 \\ 0 & 9 \end{bmatrix} + {}^{-}4 \det \begin{bmatrix} 2 & 3 \\ 0 & 9 \end{bmatrix} + {}^{+}7 \det \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix} = -48$$

$$= {}^{-}4 \det \begin{bmatrix} 2 & 3 \\ 0 & 9 \end{bmatrix} + {}^{+}5 \det \begin{bmatrix} 1 & 3 \\ 7 & 9 \end{bmatrix} + {}^{-}6 \det \begin{bmatrix} 1 & 2 \\ 7 & 0 \end{bmatrix} = -48$$

Calculate the determinant by expanding along the 3rd column.

$$\det \begin{bmatrix} a & b & c \\ d & e & 0 \\ g & h & 0 \end{bmatrix} = {}^{+}c * \det \begin{bmatrix} d & e \\ g & h \end{bmatrix} = c\,(dh - eg)$$

12.7 ♣ Matrix form of scalar equations (matrix multiplication in reverse)

Put the following sets of scalar equations into matrix form.

$a\,x + b\,y = 12$ $d\,x + e\,y = 15$ $\begin{bmatrix} a & b \\ d & e \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$	$a\,x + b\,y + c\,z = 12$ $d\,x + e\,y + f\,z = 15$ $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$	$a\,x + b\,y - 12 = 0$ $d\,x + e\,y - 15 = 0$ $\begin{bmatrix} x & y & 0 & 0 \\ 0 & 0 & x & y \end{bmatrix} \begin{bmatrix} a \\ b \\ d \\ e \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$

12.8 Optional: Solving sets of linear algebraic equations.

$$a\,x + b\,y = 1$$
$$d\,x + e\,y = 2$$

Solve for x, y.

$$x = \frac{e - 2\,b}{a\,e - b\,d} \qquad y = \frac{2\,a - d}{a\,e - b\,d}$$

$$a\,x + b\,y + c\,z = 1$$
$$2\,x + 3\,y + 4\,z = 2$$
$$2\,x + 4\,y + 6\,z = 4$$

Solve for x, y, z.

$$x = \frac{-1 - 2\,b + 2\,c}{2\,b - a - c} \qquad y = \frac{2 + 2\,a - 2\,c}{2\,b - a - c} \qquad z = \frac{-1 - 2\,a + 2\,b}{2\,b - a - c}$$

12.9 Concepts: Eigenvalues and eigenvectors

Do all the questions in Section 22.2.

12.10 ♣ Eigenvalues, determinants, and matrix algebra

- One test that the inverse of the $n \times n$ matrix A does ***not*** exist is $\det(\; A \;) = 0$
- The eigenvalues of the matrix A can be determined by setting $\det(\; -\lambda\,I + A \;) = 0$
- If an eigenvalue of the matrix A is zero, A^{-1} does ***not*** exist. **True**/**False**.

12.11 ♣ Concepts: Eigenvalues and eigenvectors

Consider the following set of algebraic equations governing the unknowns u_1, u_2, and λ.

$$\lambda\,u_1 - u_2 = 0$$
$$25\,u_1 + (\lambda - 6)\,u_2 = 0$$

or equivalently

$$\begin{bmatrix} \lambda & -1 \\ 25 & \lambda - 6 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Find "special values" of λ (called ***eigenvalues***) that allow for $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Result: $\lambda_1 = 3 + 4\,i \qquad \lambda_2 = 3 - 4\,i$

For each special value of λ determine a corresponding "special ratio" of u_2 to u_1.

Result: (These "special ratios" are called ***eigenvectors*** and c_1 and c_2 are arbitrary constants.)

For λ_1: $U_1 \triangleq \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 3 + 4\,i \end{bmatrix}$ For λ_2: $U_2 \triangleq \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = c_2 \begin{bmatrix} 1 \\ 3 - 4\,i \end{bmatrix}$

12.12 ♣ Optional: Matrix inverse with determinants and adjugate (adjunct) matrices

Calculate the following matrix inverses (the 3×3 matrix inverses are optional).

$$\text{inv}\begin{bmatrix}5\end{bmatrix} = \frac{\text{adj}\begin{bmatrix}5\end{bmatrix}}{\text{det}\begin{bmatrix}5\end{bmatrix}} = \frac{\begin{bmatrix}1\end{bmatrix}}{5} = \begin{bmatrix}0.2\end{bmatrix} \qquad \text{inv}\begin{bmatrix}a\end{bmatrix} = \begin{bmatrix}\frac{1}{a}\end{bmatrix}$$

$$\text{inv}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} = \frac{\text{adj}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}}{\text{det}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}} = \frac{\begin{bmatrix}4 & -2\\ -3 & 1\end{bmatrix}}{-2} = \begin{bmatrix}-2 & 1\\ 1.5 & -0.5\end{bmatrix}$$

$$\text{inv}\begin{bmatrix}a & b\\ c & d\end{bmatrix} = \frac{\text{adj}\begin{bmatrix}a & b\\ c & d\end{bmatrix}}{\text{det}\begin{bmatrix}a & b\\ c & d\end{bmatrix}} = \frac{\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}}{a\,d - b\,c}$$

$$\text{inv}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 0 & 9\end{bmatrix} = \begin{bmatrix}-0.9375 & 0.375 & 0.0625\\ -0.125 & 0.25 & -0.125\\ 0.7291667 & -0.2916667 & 0.0625\end{bmatrix}$$

$$\text{inv}\begin{bmatrix}a & b & c\\ d & e & 0\\ g & h & 0\end{bmatrix} = \begin{bmatrix}0 & h/(d*h-e*g) & -e/(d*h-e*g)\\ 0 & -g/(d*h-e*g) & d/(d*h-e*g)\\ 1/c & -(a*h-b*g)/(c*(d*h-e*g)) & (a*e-b*d)/(c*(d*h-e*g))\end{bmatrix}$$

12.13 Optional: Matrix computation with MotionGenesis and/or MATLAB®

Use **M**otion**G**enesis or MATLAB® to do Hw 12.1 - 12.8 (skip Hw 12.7). For example, print and submit a **M**otion**G**enesis command file named `MatrixAlgebra.txt` that uses the following commands.

GetElement	GetRow	GetColumn	GetTranspose
+ − *	GetDeterminant	GetInverse	Solve