Show work – except for ♣ fill-in-blanks (print .pdf from www.MotionGenesis.com ⇒ Textbooks ⇒ Resources).

12.1 ♣ Matrix rows and columns

Given: $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ Row 1 of M $= \begin{bmatrix} \square & \square \end{bmatrix}$ Row 2 of M $= \begin{bmatrix} \square & \square \end{bmatrix}$

$M_{2,1} = \square$ Column 1 of M $= \begin{bmatrix} \square \\ \square \end{bmatrix}$ Column 2 of M $= \begin{bmatrix} \square \\ \square \end{bmatrix}$

12.2 ♣ Matrix transpose

$$\text{Transpose} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \qquad \text{Transpose} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \square$$

12.3 ♣ Matrix addition and subtraction (+ , -)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \qquad \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} - \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} = \square$$

12.4 ♣ Scalar-matrix multiplication (*)

$$5 * \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \qquad 5 * \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \square$$

12.5 ♣ Matrix-matrix multiplication (*)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} \square \\ \square \end{bmatrix} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 3 & x \\ 5 & y \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} * \begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \qquad \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} * \begin{bmatrix} x & 3 \\ y & 5 \\ z & 7 \end{bmatrix} = \square$$

12.6 ♣ Matrix determinants

$\det \begin{bmatrix} 5 \end{bmatrix} \triangleq 5$

$\det \begin{bmatrix} a \end{bmatrix} \triangleq \square$

$$\det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \square * \square - \square * \square = -2 \qquad \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \square$$

Calculate the following determinant three ways: expand along the 1st row, 1st column, 2nd row.

$$\det \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 9 \end{bmatrix} = {+1} \det \begin{bmatrix} 5 & 6 \\ 0 & 9 \end{bmatrix} + {-2} \det \begin{bmatrix} 4 & 6 \\ 7 & 9 \end{bmatrix} + {+3} \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \square$$

$$= {+1} \det \begin{bmatrix} 5 & 6 \\ 0 & 9 \end{bmatrix} + {-4} \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + {+7} \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \square$$

$$= {-4} \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + {+5} \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + {-6} \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \square$$

Calculate the determinant by expanding along the 3rd column.

$$\det \begin{bmatrix} a & b & c \\ d & e & 0 \\ g & h & 0 \end{bmatrix} = \square * \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \square$$

12.7 ♣ Matrix form of scalar equations (matrix multiplication in reverse)

Put the following sets of scalar equations into matrix form.

$a\,x + b\,y = 12$ $d\,x + e\,y = 15$ $\begin{bmatrix} \square \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$	$a\,x + b\,y + c\,z = 12$ $d\,x + e\,y + f\,z = 15$ $\begin{bmatrix} \square \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$	$a\,x + b\,y - 12 = 0$ $d\,x + e\,y - 15 = 0$ $\begin{bmatrix} \square \end{bmatrix} \begin{bmatrix} a \\ b \\ d \\ e \end{bmatrix} = \begin{bmatrix} \square \\ 15 \end{bmatrix}$

12.8 Optional: Solving sets of linear algebraic equations.

$$a\,x + b\,y = 1$$
$$d\,x + e\,y = 2$$

Solve for x**,** y**.**

$$x = \frac{e - 2\,b}{a\,e - b\,d} \qquad y = \frac{2\,a - d}{a\,e - b\,d}$$

$$a\,x + b\,y + c\,z = 1$$
$$2\,x + 3\,y + 4\,z = 2$$
$$2\,x + 4\,y + 6\,z = 4$$

Solve for x**,** y**,** z**.**

$$x = \frac{-1 - 2\,b + 2\,c}{2\,b - a - c} \qquad y = \frac{2 + 2\,a - 2\,c}{2\,b - a - c} \qquad z = \frac{-1 - 2\,a + 2\,b}{2\,b - a - c}$$

12.9 Concepts: Eigenvalues and eigenvectors

Do all the questions in Section 22.2.

12.10 ♣ Eigenvalues, determinants, and matrix algebra

- One test that the inverse of the $n \times n$ matrix A does ***not*** exist is $\det(\square) = 0$
- The eigenvalues of the matrix A can be determined by setting $\det(\square) = 0$
- If an eigenvalue of the matrix A is zero, A^{-1} does ***not*** exist. **True/False**.

12.11 ♣ Concepts: Eigenvalues and eigenvectors

Consider the following set of algebraic equations governing the unknowns u_1, u_2, and λ.

$$\lambda\,u_1 - u_2 = 0$$
$$25\,u_1 + (\lambda - 6)\,u_2 = 0$$

or equivalently

$$\begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Find "special values" of λ (called ***eigenvalues***) that allow for $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Result: $\lambda_1 = \square$ $\qquad \lambda_2 = \square$

For each special value of λ determine a corresponding "special ratio" of u_2 to u_1.

Result: (These "special ratios" are called ***eigenvectors*** and c_1 and c_2 are arbitrary constants.)

For λ_1: $U_1 \triangleq \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ \square \end{bmatrix}$ $\qquad$ For λ_2: $U_2 \triangleq \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = c_2 \begin{bmatrix} 1 \\ \square \end{bmatrix}$