Lab 3 (associated with Hw 3): Dynamic response of a vehicle suspension system

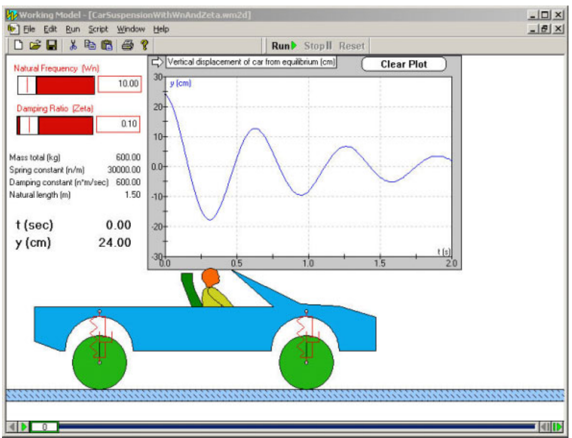
The purpose of this lab is to get a better understanding of ***2nd-order ODEs*** and to develop physical intuition for various mathematical quantities.

Lab 3.1 Effect of damping ratio (ζ) and natural frequency (ω_n) on dynamic response

For each simulation, use the plot of $y(t)$, the vertical displacement of the car versus time, to determine:

- τ_{period}, the period of vibration (if it exists)
- decayRatio $\triangleq \frac{y(t+\tau_{\text{period}}) \;-\; y_{\text{equilibrium}}}{y(t) \;-\; y_{\text{equilibrium}}}$ (if it exists)
- t_{settling}, the time required for $y(t)$ to settle within 1% of y_{ss} [the steady-state value of $y(t)$], i.e., t_{settling} is the minimum value of t such that for $t \geq t_{\text{settling}}$, $\left|y(t) - y_{\text{ss}}\right| \leq 0.01 * \left|y_{\text{ss}} - y(0)\right|$.

To begin this problem, double-click on the file `CarSuspensionWithWnAndZeta.wm2d`.



To answer each question with Working Model, click the **Reset** button (if necessary) and click and drag the sliders that control the numerical values for ω_n (measured in rad/sec) and ζ (dimensionless).

To start the simulation, click the **Run** button, and to stop it, click the **Stop** button.

(a) In the following table, record one or two significant digits for τ_{period} (in seconds), decayRatio (dimensionless), and t_{settling} (in seconds).

	ω_n=10	ω_n=20	ω_n=30
ζ=0	τ_{period} = 0.628 decayRatio = 1.0 $t_{\text{settling}} = \infty$	τ_{period} = 0.314 decayRatio = 1.0 $t_{\text{settling}} = \infty$	τ_{period} = 0.209 decayRatio = 1.0 $t_{\text{settling}} = \infty$
ζ=0.1	τ_{period} = decayRatio = t_{settling} =	τ_{period} = decayRatio = t_{settling} =	τ_{period} = decayRatio = t_{settling} =
ζ=0.2	τ_{period} = decayRatio = t_{settling} =	τ_{period} = 0.321 decayRatio = 0.277 t_{settling} = 1.151	τ_{period} = decayRatio = t_{settling} =
ζ=0.5	τ_{period} = decayRatio = t_{settling} =	τ_{period} = decayRatio = t_{settling} =	τ_{period} = 0.242 decayRatio = 0.027 t_{settling} = 0.307

(b) Based on your observations, circle the appropriate answer in the following statements.

- Increasing ω_n results in **less/more** oscillation and a **smaller/larger** period τ_{period}
- Increasing ω_n **decreases/has no effect on/increases** the decay ratio
- Increasing ω_n **decreases/has no effect on/increases** the settling time (assume $\zeta > 0$)
- Increasing ζ results in a slightly **smaller/larger** period τ_{period}
- Increasing ζ **decreases/has no effect on/increases** the decay ratio
- Increasing ζ from 0 to 0.5 **decreases/has no effect on/increases** the settling time

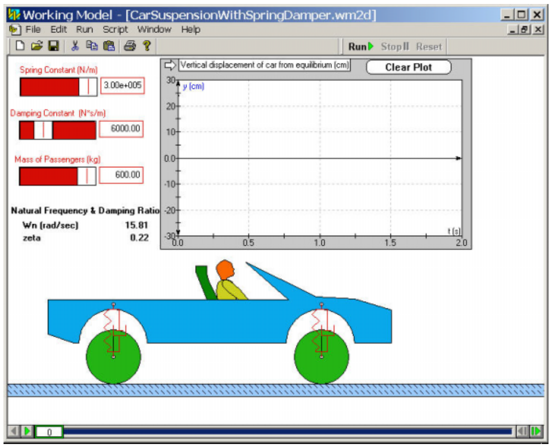
(c) For the next set of observations, use $\omega_n = 10$ rad/sec. In the row marked "Damping", write **undamped**, **underdamped**, **critically-damped**, or **overdamped**. In the row marked "t_{settling}", record the settling time (in seconds) for ζ=1.0 and ζ=2.0.

	$\zeta = 0.0$	$\zeta = 0.1$	$\zeta = 0.5$	$\zeta = 1.0$	$\zeta = 2.0$
Damping					
t_{settling}	∞	4.605	0.921		

(d) Based on your observations, circle the appropriate answers in the following statements.

- Increasing ζ from 0 to 1.0 **decreases/increases** the settling time
- Increasing ζ from 1.0 to 2.0 **decreases/increases** the settling time
- The value of ζ that produces the shortest settling time is ζ=

Lab 3.2 Effect of spring stiffness (k), damping (b), and mass (m) on dynamic response



For each simulation that follows, use the plot of $y(t)$, the vertical displacement of the car versus time, to determine the effect of k, b, and m on

- τ_{period}, the period of vibration (if it exists)
- decayRatio, the decay ratio (if it exists)
- t_{settling}, the time required for $y(t)$ to settle within 1% of its final value.

To begin this problem, double-click on the file `CarSuspensionWithSpringDamper.wm2d`.

(a) Underdamped vibrations imply a ***positive*** value of b. **True/False**.

(b) Underdamped vibrations imply a damping ratio $\zeta < 1$. **True/False**.

(c) After running as many **underdamped** simulations as necessary, fill in the following table with $--$ (decreases), $-$ (slightly decreases), **0** (no change), $+$ (slightly increases), or $++$ (increases). To fill in the last two columns, use the Working Model digital display that measures ω_n and ζ.

Effect of/on:	τ_{period}	decayRatio	t_{settling}	ω_n	ζ
Increasing k					
Increasing b					
Increasing m					

(d) Based on your observations, circle the appropriate answer in the statements that follow.

- To make a car with a higher natural frequency, use **softer/stiffer** springs
- A car's shock absorbers may be worn out if the settling time is too **short/long**
- When four large football players ride in a compact car, the ride may feel **squishy/stiff**