

Prelab	Participation	Lab
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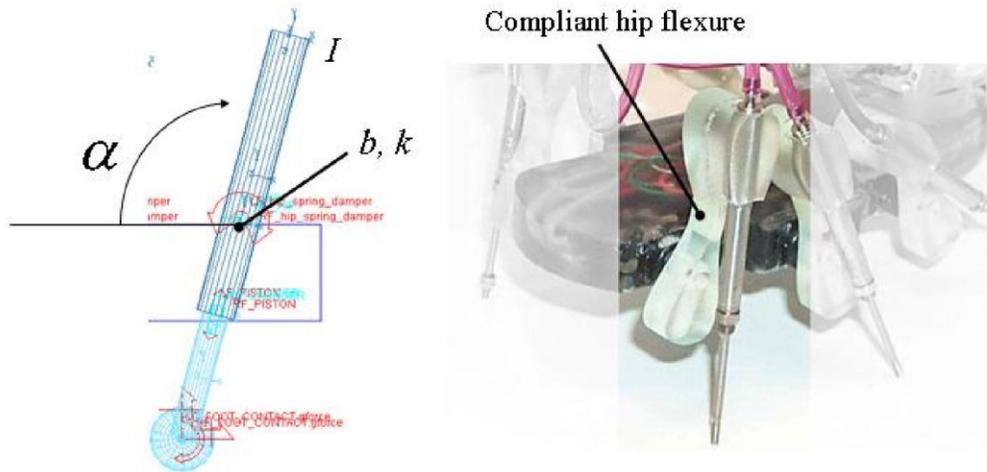
Name: _____

3 Lab 3: Rotational Systems

3.1 Introduction

The Cutkosky lab at Stanford University has been designing Sprawl robots for 10+ years. They are small, hexapedal running robots capable of speeds of over $5 \frac{\text{body-lengths}}{\text{second}}$ ($\approx 0.8 \frac{\text{m}}{\text{s}}$). They operate without feedback control, instead relying on a well-tuned physical system (set of legs) to move quickly and stably, even over obstacles or rough terrain.

During this lab you will examine the leg's rotational design and how to tune the leg for the swing phase of locomotion. During normal operation, each full stride of the robot takes 80 ms. The leg is in contact with the ground (stance phase) for about 25 ms, allowing approximately 55 ms for the leg to retract and swing forward in preparation for the next step. Experience has shown that making the legs too stiff results in a slow robot, whereas making the legs too soft results in crashes. For this lab you will be experiment with a wooden "leg" that approximates a Sprawl robot's leg.



$$I_{yy} \ddot{\theta} + b \dot{\theta} + k \theta - mgL \sin(\theta) = 0$$

3.2 PreLab: Working Model and brainstorming

1. Download the following Working Model (.wm2d) simulations.
Get these files from: www.MotionGenesis.com ⇒ [Textbooks](#) ⇒ [Resources](#)

MetronomeWnZeta.wm2d	MetronomeIBK.wm2d
MetronomeNonlinearAngle.wm2d	

2. Run the Working Model simulations.
Record results on the [Working Model PreLab](#) (.pdf on www.MotionGenesis.com).

3.3 Experimental

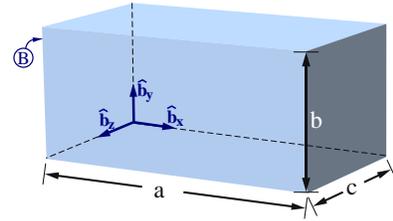
The system that you will be evaluating rotates using a compliant joint. The joint was designed and fabricated (from a soft grade of urethane) to be used as a leg flexure for Sprawlita. The leg flexure can be modeled as a pin joint with a built-in rotational spring and damper. It will be your job to experimentally determine the effective stiffness and damping of this joint.

3.3.1 Analytical determination of moment of inertia of a solid block

The moment of inertia of a rigid body about a point is defined with a line (axis). Changing the specified point on the body or the line's orientation changes the associated moment of inertia.

The moment of inertia of a uniform rectangular block about the y-axis passing through the block's center of mass is:

$$I_{yy} = \frac{1}{12} m (a^2 + c^2)$$



Mass of block	$m = \underline{\hspace{2cm}}$ kg	Note: Measure the object's mass with a scale	
Length of block	$a = \underline{\hspace{2cm}}$ m	Height of block	$b = \underline{\hspace{2cm}}$ m
		Width of block	$c = \underline{\hspace{2cm}}$ m
Calculate moment of inertia:		$I_{yy} = \underline{\hspace{2cm}}$ kg m ²	

Does I_{yy} depend on the block's height? **Yes/No.**

Explain: Moment of inertia depends on mass-distribution **away from/along** a line.

3.3.2 Methods for determining moments of inertia

Moments of inertia play a central role in the rotational motions of objects. List a few methods for determining (e.g., calculating, measuring, or approximating) moments of inertia of large irregularly-shaped objects such as an airplane or automobile.

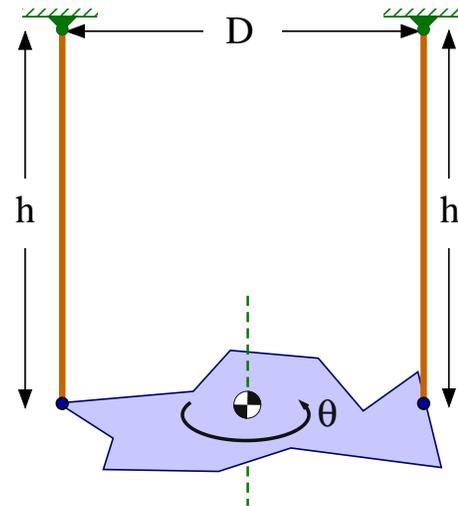
- Compute:
- Experiment:
- Approximate:

3.3.3 Experimental apparatus

- h Length of each suspension wire
- D Distance between (2 parallel) wires
- mg Weight of the object
- I_{yy} Mass moment of inertia about y-axis
- T Tension in string
- θ Angle of rotation

As the object rotates in the horizontal plane, it **moves up and down**. The linearized equation of motion is^a

$$\ddot{\theta} + \frac{mgD^2}{4I_{yy}h} \theta = 0$$



Trifilar pendulum are used to hang aircraft and measure moments of inertia. 6 experiments lead to 6 equations for the aircraft's inertia properties.

^a One way to form the linearized equation of motion is to first generate the nonlinear equation of motion (not so simple) and then linearize the nonlinear equation about $\theta = 0$, $\dot{\theta} = 0$, $\ddot{\theta} = 0$.

Using our knowledge of ODEs and rearranging to solve for the moment of inertia gives

$$\omega_n = \sqrt{\frac{m g D^2}{4 I_{yy} h}} \Rightarrow \tau_{\text{period}} = \frac{2\pi}{\omega_n} = 2\pi \sqrt{\frac{4 I_{yy} h}{m g D^2}} \Rightarrow I_{yy} = \frac{m g D^2}{16 \pi^2 h} \tau_{\text{period}}^2$$

Record measurements of the bifilar pendulum to the right. Hang the block on two strings (ensure they are **equal length** and **parallel**). Use a stopwatch to determine the pendulum's period of vibration. Provide an initial **small angle** of $\theta(t=0) \approx 30^\circ$.

Mass of block	$m =$ _____ kg
Length of wires	$h =$ _____ m
Distance between wires	$D =$ _____ m
Period of vibration	$\tau_{\text{period}} =$ _____ sec
Moment of inertia	$I_{yy} =$ _____ kg m ²

My predicted and measured values agree to within (circle one)

Less than 0.1%	Less than 1%	Less than 10%	Less than 50%	More than 50%
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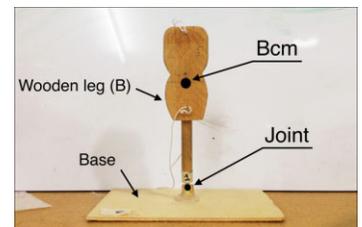
Note: If you got more than 10% deviation, you probably did something wrong.

Repeat the bifilar experiment for the wooden leg

Using the parallel axis theorem and the moment of inertia determined via a bifilar pendulum experiment, calculate leg B 's moment of inertia about the joint.

Hint: Use the parallel axis theorem, B 's moment of inertia about its center of mass B_{cm} , the mass of the block, and the distance from the pivot to B_{cm} .

$$I_{yy}^{B/\text{Joint}} = I_{yy}^{B/B_{\text{cm}}} + m * \text{distance}^2 = \text{_____ kg m}^2$$



Mass of leg	$m =$ _____ kg
Length of wires	$h =$ _____ m
Distance between wires	$D =$ _____ m
Period of vibration	$\tau_{\text{period}} =$ _____ sec
Moment of inertia	$I_{yy} =$ _____ kg m ²

3.3.4 Characterization of the Compliant Joint

Carefully assemble the leg by sliding the joint into the base. Tape the accelerometer to the top of the leg. When you displace the leg, please do so gently. Keep in mind that our calculations use a **small angle approximation**. For this part of the lab we again use an ADXL 311 accelerometer from Analog Devices. The data acquisition proceeds in a manner similar to previous labs, sampling again at **1 kHz**.

1. Fix the leg to the wooden base (the system is an inverted pendulum).
2. Tape the accelerometer to the leg's most distal surface (top of the leg) so the cables are parallel to the axis of rotation.
3. Power the Arduino by plugging-in the USB cable (between the board and computer):
4. Click the magnifying glass button (or type Ctrl+Shift+m) to open the serial monitor
5. On the serial monitor screen, a menu should appear. Enter "a" to start reporting data.
6. Have one group member pull the leg slightly to one side and then release.
7. Enter "a" or "r" to stop recording data after the leg has stopped.
8. Plot the data (e.g., using Excel, MATLAB[®], or PlotGenesis).
9. Email the data files and/or graphs to yourself and your group members.
10. **Print and submit** a graph of the leg's acceleration ($\frac{m}{s^2}$) vs. time (sec) with your lab. See Section 3.3.6 for accelerometer data acquisition information.
11. Ensure the power to the board is off and the setup is neat for the next lab.

3.3.5 Questions

Shown right is the ODE for θ when the small angle approximation $\sin(\theta) \approx \theta$ is used. Form symbolic expressions for θ 's natural frequency ω_n and damping ratio ζ . Next, use the graph you generated in Section 3.3.4 to determine numerical values for τ_{period} (period of vibration) and decay ratio.

$$I_{yy} \ddot{\theta} + b \dot{\theta} + (k - mgL) \theta = 0$$

$$\omega_n = \sqrt{\frac{\text{[]}}{\text{[]}}} \quad \zeta = \frac{\text{[]}}{\text{[]}}$$

$$\tau_{\text{period}} = \text{[]} \text{ sec} \quad \text{decayRatio} = \text{[]} \text{ NoUnits}$$

Show calculations that use the lab data to determine numerical values for ω_n and ζ . Next, show how to calculate numerical values for this system's physical parameters b , k .

$$\omega_n = \text{[]} \frac{\text{rad}}{\text{sec}} \quad \zeta = \text{[]} \text{ no units}$$

$$b = \text{[]} \text{ N m sec} \quad k = \text{[]} \frac{\text{Nm}}{\text{rad}}$$

Note: I_{yy} was determined earlier.

Turning the leg upside down: increases/decreases/has no affect on ω_n
 increases/decreases/has no affect on ζ .

† Using your value for the wooden leg's mass moment of inertia (I_{yy}) and experimentally determined value of the flexure stiffness k determine the damping constant b , needed to ensure a settling time of less than 55 ms.

$$b = \text{_____} \text{ N m sec}$$

3.3.6 Optional Info: Experimental estimation with an accelerometer

We use several pieces of equipment to measure and record the cart's horizontal acceleration, namely, we use an accelerometer, a microprocessor, a transceiver, and a computer.⁸

- **Accelerometer: ADXL 311 from Analog Devices**

The accelerometer is mounted on a cart and measures acceleration in up to three directions (we use data from only one direction). The accelerometer is relatively small and lightweight as compared to the cart - so its effect on the acceleration of the cart is negligible. The accelerometer's output signal is a linear $0.3 \frac{\text{volts}}{g}$ signal over a range of $\pm 2 g$.⁹ The accelerometer is designed to output 2.5 volts when there is *no* acceleration, but there is some variation from one accelerometer to the next. This is why we measured the output of the accelerometer in Section 4.2.1.

$$v_{\text{accelerometer}} \approx v_{0g} + \left(\frac{0.3 \text{ volts}}{1 g} \right) \ddot{x}$$

- **Breadboard signal processing**

The accelerometer's signal is filter by a low-pass filter to remove high-frequency noise in the signal. Op-amps are used as a buffer to supply additional current and avoid unwanted voltage drops.

- **Arduino UNO microprocessor:**

The microprocessor's A/D port receives an analog voltage signal from the breadboard in a specified range (i.e., continuous voltages from 0 volts to 5 volts). The A/D port samples the analog signal at 1000 Hz (i.e., at 1 ms intervals). The 10 bit A/D converter on the microprocessor changes the 0 to 5 volt analog signal to bits (ones and zeros) that represent $2^{10} = 1024$ integer values (e.g., 5 volts converts to 1024 and 2.5 volts converts to 512). The Arduino transmits this data to the computer via serial communication.

- **Computer:**

The computer receives bits from its serial port and uses the Arduino software program to translate the bits to integer numbers which are then printed to the screen. The numbers displayed on the computer screen are integers from 0 to 1024.

⁸Most accelerometers do not come assembled with a microprocessor, transceiver, and computer.

⁹NIST (National Institute of Standards and Technology) defines $1 g$ as exactly 9.80665 m/s^2 .