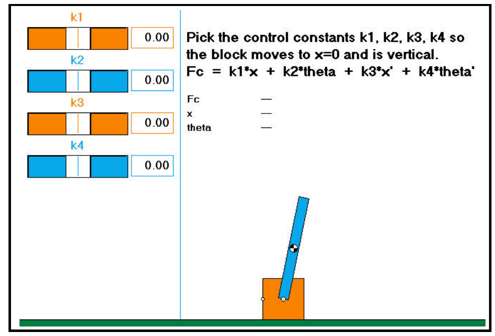
Lab 14 (associated with Hw 14): Feedback control of an inverted pendulum on a cart

This lab designs a control system to regulate the motion of a dynamic system. To pick control constants, Lab 14.1 uses physical insight (guess and check) whereas Lab 14.2 applies your mathematical skills.

Lab 14.1 Feedback control of inverted pendulum on cart with intuition.

To begin this problem, double-click on the file `InvertedPendulumIntuition.wm2d`.

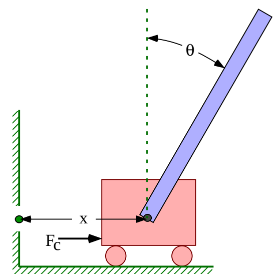


To answer each question with Working Model, click the **Reset** button (if necessary) and click and drag the sliders to choose feedback control constants for k_1, k_2, k_3, k_4, that control the motion of this system.

To start the simulation, click the **Run** button, and to stop it, click the **Stop** button.

To control the position x of the cart and the orientation θ of the inverted pendulum, appropriate values must be chosen for the feedback control constants k_1, k_2, k_3, k_4, for the feedback control law

$$F_c \;=\; k_1\,x \;+\; k_2\,\theta \;+\; k_3\,\dot{x} \;+\; k_4\,\dot{\theta}$$



Based on your observations when you run simulations and your physical intuition, circle appropriate answers in the following statements.

(a) Try to control just the cart's position so $x(t) \approx 0$ by trying various values of k_1 and k_3. Do not also try to control θ (set $k_2 = k_4 = 0$).

- Using intuition, it is **easy/difficult/impossible** to control only $x(t)$.
- With $k_1 > 0$ and $k_3 > 0$ the response for $x(t)$ is **stable/neutrally stable/unstable**.
- With $k_3 = 0$, making k_1 more negative yields **less/more** oscillation and a **smaller/larger** period.
- With $k_3 = -100$, making k_1 more negative **decreases/increases** the settling time of $x(t)$.
- Making k_1 more negative results in a **slower/faster** speed of response of $x(t)$.
- With $k_1 = -80$, changing k_3 from -20 to -40 **decreases/increases** the settling time of $x(t)$.
- With $k_1 = -80$, changing k_3 from -100 to -200 **decreases/increases** the settling time of $x(t)$.
- With $k_1 = -80$ and $k_3 = -20$, $x(t)$ seems **underdamped/critically-damped/overdamped**.
- With $k_1 = -80$ and $k_3 = -200$, $x(t)$ seems **underdamped/critically-damped/overdamped**.

(b) Try to control just the inverted pendulum so $\theta(t) \approx 0$ by trying various values of k_2 and k_4. Do not also try to control x (set $k_1 = k_3 = 0$).

- Using intuition, it is **easy/difficult/impossible** to control $\theta(t)$.
- With $k_4 = 0$, the value of k_2 that results in $\theta(t)$ being neutrally stable is -200/-100/**100/200**.
- With $k_2 = 200$ and $k_4 < 0$, $\theta(t)$ is **stable/neutrally stable/unstable**.
- With $k_2 = 200$ and $k_4 = 10$, $\theta(t)$ seems **underdamped/critically-damped/overdamped**.
- With $k_2 = 200$ and $k_4 = 50$, $\theta(t)$ seems **underdamped/critically-damped/overdamped**.
- With $k_2 = 200$ and $k_4 = 90$, $\theta(t)$ seems **underdamped/critically-damped/overdamped**.

(c) Using intuition, take your best ***guess*** at the signs of k_1, k_2, k_3, k_4 (e.g., circle $-$ for negative or $+$ for positive) so F_c controls both $x(t) \approx 0$ and $\theta(t) \approx 0$.

$$F_c \;=\; k_1\,x \;+\; k_2\,\theta \;+\; k_3\,\dot{x} \;+\; k_4\,\dot{\theta}$$

Result: $k_1 =$ **+ 0 −** $\quad k_2 =$ **+ 0 −** $\quad k_3 =$ **+ 0 −** $\quad k_4 =$ **+ 0 −**

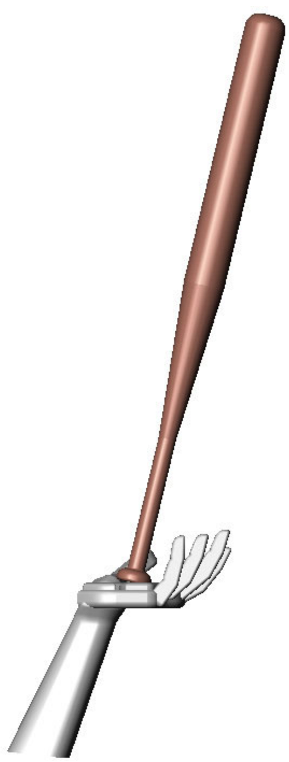
(d) Use your intuition to control ***both*** $x(t) \approx 0$ and $\theta(t) \approx 0$. This seems **easy/difficult/impossible**.

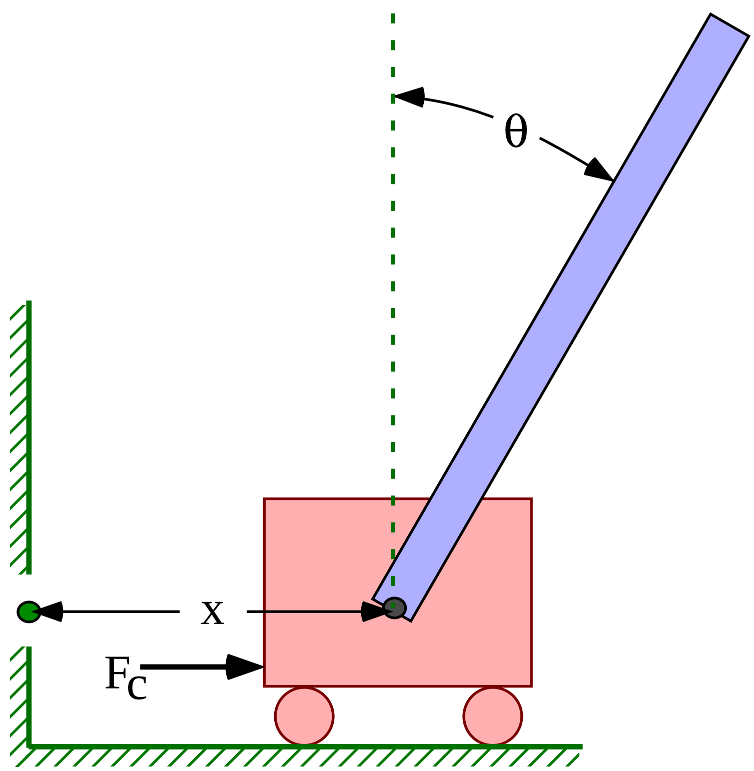
(e) Try the Homework 14.3 feedback control constants of $k_1 = 10$, $k_2 = 200$, $k_3 = 11$, $k_4 = 35$.
With these values, the system's motion is **stable/neutrally stable/unstable**.
All/some/none of these values of k_1, k_2, k_3, k_4 have the same signs as my guess.
These instruct F_c to push **left/right** when $x = 1$, $\theta = \dot{x} = \dot{\theta} = 0$.
This **does/does not** make intuitive sense to me.

(f) Reset the simulation by clicking the **Reset** button. With your new insights for the signs of k_i ($i = 1, ..., 4$), try (for two minutes) to choose new values of k_i to stabilize the system.
Result: $k_1 =$ ____ $k_2 =$ ____ $k_3 =$ ____ $k_4 =$ ____

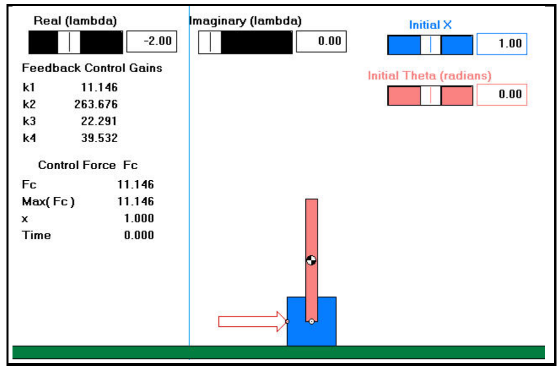
(g) The point of Lab 14.1 is to (circle one or more)

- Frustrate you.
- Help you appreciate the value of mathematics when intuition fails.
- Help you realize that the behavior of ***coupled*** ODEs can be significantly more difficult to understand than that of ***uncoupled*** ODEs.





Lab 14.2 Feedback control of inverted pendulum on cart with pole placement.
The point of this problem is to control x and θ using the pole placement methods of Homework 14.4. To begin this problem, double-click on the file `InvertedPendulumPolePlacement.wm2d`.



To answer each question with Working Model, click the **Reset** button (if necessary) and click and drag the sliders that control the system's pole locations and motion.

To start the simulation, click the **Run** button, and to stop it, click the **Stop** button.

In Homework 14.4, the state matrix Y had a solution

$$Y(t) \triangleq \begin{bmatrix} x \\ \theta \\ \dot{x} \\ \dot{\theta} \end{bmatrix} = c_1\, U_1\, e^{\lambda_1 t} + c_2\, U_2\, e^{\lambda_2 t} + c_3\, U_3\, e^{\lambda_3 t} + c_4\, U_4\, e^{\lambda_4 t}$$

c_i are constants that depend on initial values $(i = 1, ..., 4)$
U_i are 4×1 matrices of constants (eigenvectors) $(i = 1, ..., 4)$
λ_i are constants (poles/eigenvalues). $(i = 1, ..., 4)$

(a) Set $x(0) = 1$ m and $\theta(0) = 0$. Try various values for Real(λ) and Imag(λ).
Note: Pole locations were chosen so $\lambda_1 = \lambda_3 =$ Real(λ) + Imag(λ) i and $\lambda_2 = \lambda_4 =$ Real(λ) $-$ Imag(λ) i.

λ	Stable (based on λ)	Oscillatory (based on λ)	Stable response
-1	**stable/neutrally stable/unstable**	**Yes/No**	**Yes/No**
-2	**stable/neutrally stable/unstable**	**Yes/No**	**Yes/No**
-3	**stable/neutrally stable/unstable**	**Yes/No**	**Yes/No**
-4	**stable/neutrally stable/unstable**	**Yes/No**	**Yes/No**
$0 + 1\,i$	**stable/neutrally stable/unstable**	**Yes/No**	**Yes/No**
$-1 + 1\,i$	**stable/neutrally stable/unstable**	**Yes/No**	**Yes/No**
$-1 + 2\,i$	**stable/neutrally stable/unstable**	**Yes/No**	**Yes/No**
$-1 + 3\,i$	**stable/neutrally stable/unstable**	**Yes/No**	**Yes/No**
$-1 + 4\,i$	**stable/neutrally stable/unstable**	**Yes/No**	**Yes/No**

(b) The fastest time to stabilize is achieved with $\lambda =$ **-1 -2 -3 -4** (circle one).
The smallest maximum force that stabilizes the response is achieved with $\lambda =$ **-1 -2 -3 -4**.

(c) Increasing Imag(λ) results in **less/more** oscillation.
Making Real(λ) more negative **decreases/increases** the settling time.

(d) When $x(0) = 0.25$, $\theta(0) = 0$, and $\lambda = -1 + 4\,i$, the response is **stable/unstable** which is different than what was found when $x(0) = 1$. The difference in stability is due to the fact that the system's ODEs are __________.

(e) Pick Real(λ) and Imag(λ) so the system's response is stabilized when $x(0) = 2$ and $\theta(0) = 0$.
Result:

$$\lambda = \underline{\quad\quad} + \underline{\quad\quad}\, i$$

Now, try the following initial values of θ: 0.2 rads, 0.4 rads, 0.8 rads. As $\theta(0)$ increases, the system seems **less/more** stable. This happens because the system's ODEs are __________.