Lab: MIPSI for a dynamic system (one lab per group)

PreLab: Brainstorm a physical system with an interesting question (not a Ph.D. dissertation), e.g., system identification, design, stability, control, . . .

1. Picture of you and your team
2. Picture/sketch of your system
3. Question you would like to answer

The objective of a **MIPSI** is to ask and answer a question for a system of your own choosing.
The **2-3** page report **clearly communicates** its question, technical information, and answer.[3]
Attach multi-page appendix with all computer files (symbolic/numeric) and supporting calculations and figures.

- **Question** Ask an interesting dynamic systems question e.g., in biology (exercise, muscle activation, nature), mechanics (vehicles, boat, swing, yo-yo, home appliances, pump, motor), aerospace (aircraft, helicopter, balloon), fluid mechanics, heat transfer, thermodynamics, economics, electronics, chemistry, control-systems, vibrations, modal analysis, input shaping, robotics, mechatronics, haptics, machine design, biomechanics, molecular dynamics, financial modeling.

- **Model.** Draw one or more sketches. Use engineering insight to determine the relevant system components and simplify the model. Report modeling assumptions/approximations.

  Type a **short** problem statement starting with "The following figure shows", and then describe all objects. Ensure all physical objects are clearly labeled on sketches and described in text (e.g., name and label airplane A, book B, point P, etc).

- **Identifiers (symbols and values).**

  Include a table of relevant scalar identifiers with four columns labeled:

Quantity	Identifier	Type	Value

  (**estimate** numbers for constants and initial values).
  Show relevant scalar identifiers on the sketch(es) without clutter.

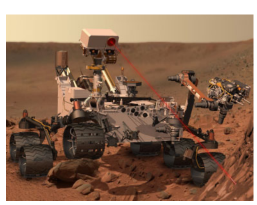
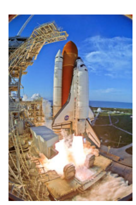
- **Physics.** Form equations relating the identifiers to system behavior.
  Report calculations for forming the system's governing ODEs (attach long calculations in an appendix).
  Attach long calculations and MATLAB®/**M**otion**G**enesis codes in an appendix.

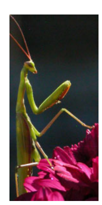
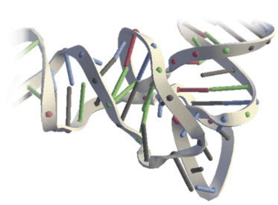
- **Simplify and Solve.** If helpful, make small angle or linear approximations [e.g., $\sin(\theta) \approx \theta$].
  Discuss the process for solving for the unknown identifiers [e.g., numerical solution via **M**otion**G**enesis, MATLAB®, or WolframAlpha or analytical (closed-form) solutions].

- **Interpret (design and control).** Answer your question with results easily interpreted by a non-technical person (use words, numbers, plots, video, etc., with descriptive text **adjacent** each plot).
  Optional: Build the physical system, validate the analysis, physical demonstration, video.

Model	5%	**Cover-page**: Team **picture** (with names), system **picture**, your **question** and its **answer**.
Identifiers	15%	Detailed modeling assumptions and comprehensible schematics (preferably with photo). Precise description of all physical objects.
Physics	10%	Concise accurate tabular description of all **scalar** symbols.
Simplify and solve	45%	Correct MG road-map and/or high-level summary of calculations. Correct analysis. **Short (2-3 pg.)**, solid report. Appendix of calculations.
Interpret and design	15%	Interpret: Relevant text interspersed with relevant plots.
	10%	On-schedule. Met with instructor. Technical difficulty, demo/video, interesting problem.

Communicate!

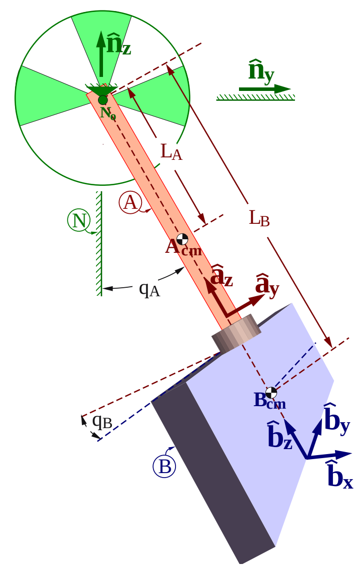




[3]For ideas, see videos at www.MotionGenesis.com ⇒ Textbooks ⇒ Resources or www.YouTube.com.

Motivating example: Babyboot

Modeling



The figure to the right is a schematic representation of a swinging babyboot attached by a shoelace to a rigid support. The mechanical model of the babyboot consists of a thin uniform rod A attached to a fixed support N by a revolute joint, and a uniform plate B connected to A with a second revolute joint so that B can rotate freely about A's axis.

Note: The revolute joints' axes are ***perpendicular***, not parallel.

Modeling considerations

- The plate, rod, and support are rigid.
- The revolute joints are frictionless.
- There is no slop or flexibility in the revolute joints.
- Earth is a Newtonian reference frame.
- Air resistance is negligible.
- Forces due to Earth's gravitation are uniform and constant.
- Other distance forces (electromagnetic and gravitational) are negligible.

Identifiers

Right-handed sets of unit vectors $\hat{\mathbf{n}}_x$, $\hat{\mathbf{n}}_y$, $\hat{\mathbf{n}}_z$; $\hat{\mathbf{a}}_x$, $\hat{\mathbf{a}}_y$, $\hat{\mathbf{a}}_z$; and $\hat{\mathbf{b}}_x$, $\hat{\mathbf{b}}_y$, $\hat{\mathbf{b}}_z$ are fixed in N, A, and B, respectively, with $\hat{\mathbf{n}}_x = \hat{\mathbf{a}}_x$ parallel to the revolute axis joining A to N, $\hat{\mathbf{n}}_z$ vertically upward, $\hat{\mathbf{a}}_z = \hat{\mathbf{b}}_z$ parallel to the rod's long axis (and the revolute axis joining B to A), and $\hat{\mathbf{b}}_z$ perpendicular to plate B.

Quantity	Symbol	Type	Value
Earth's gravitational constant	g	Constant	9.81 m/s^2
Distance between N_o and A_{cm}	L_A	Constant	7.5 cm
Distance between N_o and B_{cm}	L_B	Constant	20 cm
Mass of A	m^A	Constant	0.01 kg
Mass of B	m^B	Constant	0.1 kg
A's moment of inertia about A_{cm} for $\hat{\mathbf{a}}_x$	I^A	Constant	0.05 kg∗cm^2
B's moment of inertia about B_{cm} for $\hat{\mathbf{b}}_x$	I^B_x	Constant	2.5 kg∗cm^2
B's moment of inertia about B_{cm} for $\hat{\mathbf{b}}_y$	I^B_y	Constant	0.5 kg∗cm^2
B's moment of inertia about B_{cm} for $\hat{\mathbf{b}}_z$	I^B_z	Constant	2.0 kg∗cm^2
Angle from $\hat{\mathbf{n}}_z$ to $\hat{\mathbf{a}}_z$ with $+\hat{\mathbf{n}}_x$ sense	q_A	Dependent variable	varies
Angle from $\hat{\mathbf{n}}_z$ to $\hat{\mathbf{a}}_z$ with $+\hat{\mathbf{n}}_x$ sense	q_B	Dependent variable	varies
Time	t	Independent variable	varies

Physics

Answers at www.MotionGenesis.com ⇒ Get Started ⇒ Chaotic Pendulum (Babyboot).

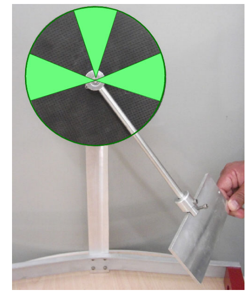
The ODEs (ordinary differential equations) governing the motion of this mechanical system are[2]

$$\ddot{q}_A \;=\; \frac{2\,\dot{q}_A\,\dot{q}_B\,\sin(q_B)\,\cos(q_B)\,\left(\mathrm{I}^B_x - \mathrm{I}^B_y\right) \;-\; \left(m^A L_A + m^B L_B\right) g \sin(q_A)}{\mathrm{I}^A \,+\, m^A L_A^2 \,+\, m^B L_B^2 \,+\, \mathrm{I}^B_x \cos^2(q_B) \,+\, \mathrm{I}^B_y \sin^2(q_B)}$$

$$\ddot{q}_B \;=\; \frac{\text{-}\dot{q}_A{}^2\,\sin(q_B)\,\cos(q_B)\,(\mathrm{I}^B_x - \mathrm{I}^B_y)}{\mathrm{I}^B_z}$$

[2] Four methods for forming equations of motion are: ***Free-body diagrams*** of A and B (which is inefficient as it introduces up to 10 unknown force/torque measures); D'Alembert's method (***road maps*** of Section 20.8) which efficiently forms the two equations shown for $\ddot{q}_A$ and $\ddot{q}_B$ (but require a clever selection of systems, points, and unit vectors); ***Lagrange's equations*** (an energy-based method that automates D'Alembert's cleverness); ***Kane's equations*** (a modern efficient blend of D'Alembert and Lagrange).

Simplify and solve

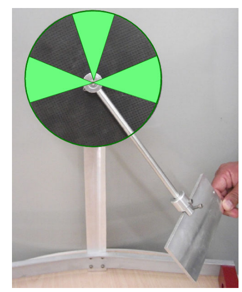


```
Variable  qA'', qB''       % Angles and first/second time-derivatives.
%--------------------------------------------------------------------
qA'' = 2*( 508.89*sin(qA) - sin(qB)*cos(qB)*qA'*qB' ) / (-21.556 + sin(qB)^2)
qB'' = -sin(qB)*cos(qB)*qA'^2
%--------------------------------------------------------------------
Input  tFinal = 10 sec, integStp = 0.02 sec, absError = 1.0E-07
Input  qA = 90 deg,     qB = 1.0 deg,        qA' = 0.0 rad/sec,  qB' = 0.0 rad/sec
OutputPlot  t sec,  qA degrees,  qB degrees
%--------------------------------------------------------------------
ODE() solveBabybootODE
Quit
```

Interpret

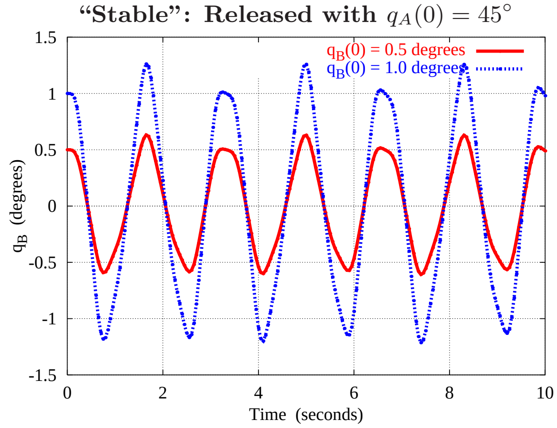
The solution to these differential equations reveals this simple system has strange, non-intuitive motion.[3] For certain initial values of q_A, the motion of plate B is well-behaved and "stable". Alternately, for other initial values of q_A, B's motion is **"chaotic"** – meaning that a small variation in the initial value of q_B or numerical integration inaccuracies lead to dramatically different results (these ODEs are used to test the accuracy of numerical integrators – the plots below required a numerical integrator error of absError $= 1 \times 10^{-7}$).

The following chart and figure to the right shows this system's regions of stability (white) and instability (**green**). Notice the **"chaotic"** plot below shows q_B is ***very*** sensitive to initial values. A 0.5° change in the initial value of $q_B(0)$ results in more than a 2000° difference in the value of $q_B(t = 10)$!

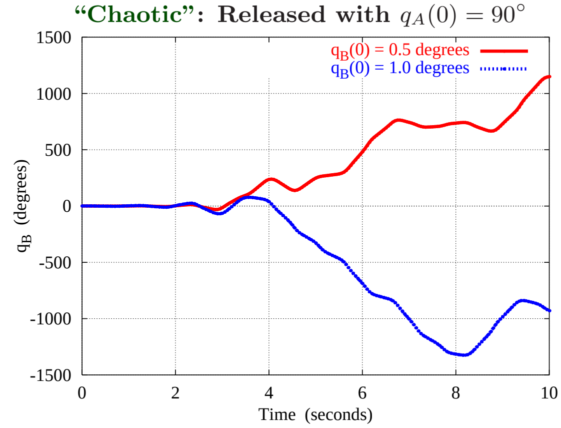


Initial value of q_A			Stability	
0°	$\le q_A(0) \le$	71.3°	Stable	(white)
71.4°	$\le \mathbf{q_A(0)} \le$	**111.77°**	**Unstable**	**(green)**
111.78°	$\le q_A(0) \le$	159.9°	Stable	(white)
160.0°	$\le \mathbf{q_A(0)} \le$	**180.0°**	**Unstable**	**(green)**

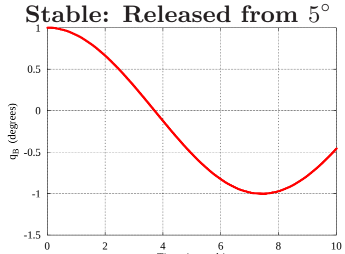
"Stable": Released with $q_A(0) = 45°$



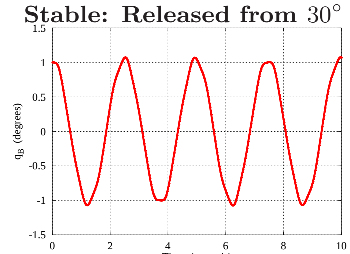
"Chaotic": Released with $q_A(0) = 90°$



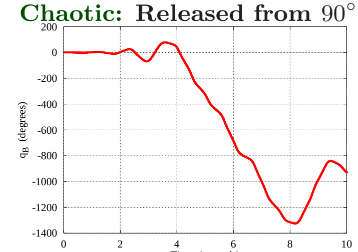
Stable: Released from 5°



Stable: Released from 30°



Chaotic: Released from 90°



[3]More information about this problem is in "Mechanical Demonstration of Mathematical Stability and Instability", *International Journal of Engineering Education (Journal of Mechanical Engineering Education)*, Vol. 2, No. 4, 1974, pp. 45-47, by Thomas R. Kane. Or visit www.MotionGenesis.com ⇒ Get Started ⇒ `Chaotic Pendulum (Babyboot)`.