Prelab	Participation	Lab
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Name: ____________________

6 Lab: Motor Constants

Motors actuate and control dynamic systems. Understanding motors help improve system design and performance. Two quantities that characterize a motor's function are the ***motor torque constant*** k_m and the ***motor voltage constant*** k_v that appear in the following formulas

T_m is motor torque.
i_m is current through the motor.

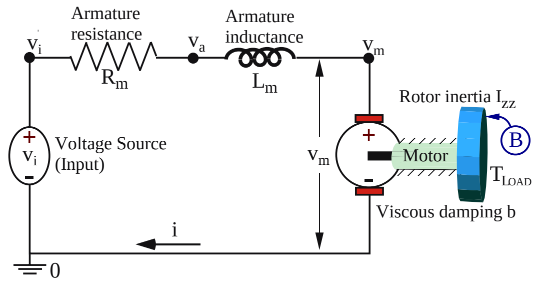
$$T_m = k_m \, i_m \qquad\qquad v_m = k_v \, \omega_m$$

v_m is motor back-EMF voltage.
ω_m is motor angular speed.

The values of k_m and k_v are usually found on an experimentally-determined ***motor specification sheet*** (e.g., from the motor's manufacturer). You will experiment to determine their values.

6.1 Experimental

The schematic shown right shows an ideal DC motor. This motor model includes the voltage supplied to the motor (v_i), the motor's coil resistance (R_m), the motor's inductance (L_m), and the motor's back-EMF (v_m). The ODE that relates the motor's angular speed ω_m to the input voltage v_i is[a]



[a]This electro-mechanical ODE is in the homework.

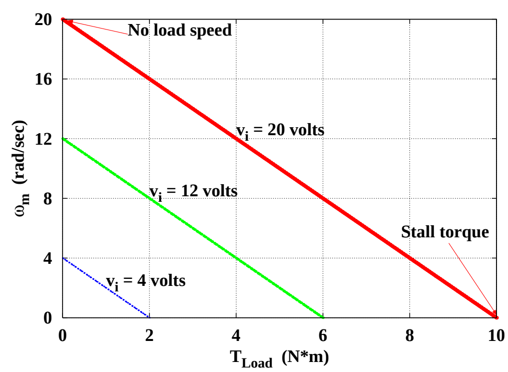
$$L_m \, I_{zz} \, \ddot{\omega}_m \;+\; (L_m \, b + R_m \, I_{zz}) \, \dot{\omega}_m \;+\; (R_m \, b + k_m \, k_v) \, \omega_m \;=\; k_m \, v_i \;-\; R_m \, T_{Load} \;-\; L_m \, \dot{T}_{Load}$$

$$\text{When } T_{Load} \text{ and } v_i \text{ are constant:} \qquad (R_m \, b + k_m \, k_v) \, \omega_{m\,ss} \;=\; k_m \, v_i \;-\; R_m \, T_{Load}$$

Solve for the linear relationship between the motor's steady-state angular speed ω_m and a constant value of T_{Load} (when v_i is constant).

$$\omega_{m\,ss} = \boxed{} \; v_i \; - \; \boxed{} \; T_{Load}$$

Graphed right is this line, showing that increasing the input voltage v_i increases the line's offset.

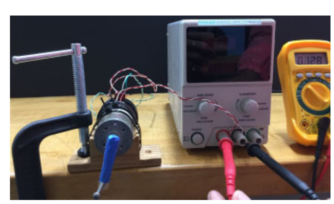


Note: The slope of this line is k_{gradient}, the motor's ***speed-torque gradient constant***.

6.1.1 Stall torque

Write the linear relationship for T_{Load} when both v_i and ω_m are **constant**. Solve for k_m in terms of the steady-state value of T_{Load} when v_i is **constant** and the motor is ***stalled***, i.e., $\omega_m(t) = 0$.

$$T_{Load}\Big|_{\substack{\text{constant } v_i \\ \text{constant } \omega_m}} = \frac{\boxed{}}{\boxed{}} \omega_m + \frac{\boxed{}}{\boxed{}} v_i \quad \Rightarrow \quad k_m = \frac{\boxed{}}{\boxed{}} T_{Load}$$

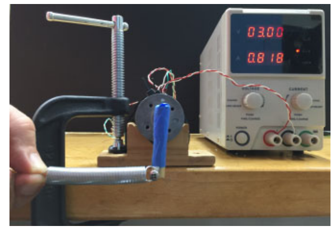


While the motor is off, use the multi-meter to measure the motor coil resistance at several angular positions.

Average Resistance: $R_m \approx$ ______ Ohms

First, experimentally determine the spring constant (provided in lab/assume linear spring) by hanging a known weight and measuring displacement. Next, attach the spring to the motor's lever arm (provided in lab). Turn the motor on (with v_i from 3 to 5 volts). stall the motor by stretching the spring (orient the spring so it is perpendicular to the lever arm), measure the spring displacement, and complete the table below to calculate ***stall torque***. Record the length of the moment arm.

Power-supply knob Approx. v_i	Multimeter Measured v_i	Spring Stretch	Spring Force	Spring Torque	k_m
3.0					
4.0					
5.0					
				Average k_m	



Do not stall the motor for long or it will overheat and burn out.

6.1.2 No-load angular speed

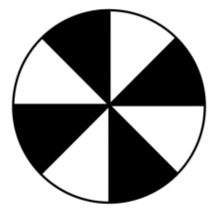
Assuming $b \approx 0$, solve for k_v in terms of the steady-state value of ω_m when v_i is **constant** and there is ***no load*** on the motor, i.e., $T_{Load}(t) = 0$.

$$k_v = \frac{\quad}{\quad}$$

6.1.3 Estimation with an Encoder and Arduino remove lever arm from motor

We use an several pieces of equipment to measure and record the motor's angular speed,

The optical quadrature **encoder** measures motor rotational speed by detecting alternating light and dark patterns on a disk. For example, the encoder on the right shows 8 transitions (from light to dark or vice-versa). A quadrature encoder can detect both angular speed **and** direction. Our encoder has 1000 transitions (500 black sections and 500 white sections) and counts 1000 $\frac{\text{tics}}{\text{rev}}$.



Arduino: Use lab6.ino to measure the steady-state constant value of ω_m.

Remove the lever arm from the motor. Connect the motor to the power supply. Using a motor without load, measure the steady-state value of ω_m for v_i from 0 to 6 volts.

Power-supply knob Approx. v_i	Multimeter Measured v_i	Angular speed ω_m (rpm)	Angular speed ω_m (rad/sec)	k_m
2.0				
4.0				
6.0				
			Average k_v	

6.1.4 Comparison and verification

Calculate the percent error in estimates for k_m and k_v.

Note: The definition of "volt" unit leads to (for an ideal DC motor) $k_m = k_v$ when SI units are used.

$$\frac{k_m - k_v}{k_v} * 100 = \quad \%$$

Why might k_m and k_v differ by more than 10%? List 2^+ sources of errors in your experiments.

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Optional: † In Lab 1, we found these motors had Coulomb friction. Based on the data just taken, better estimate k_v by first estimating a friction torque T_f in units of N m.

(Hint: There are two unknowns T_f and k_v - hence you may need multiple experiments).