

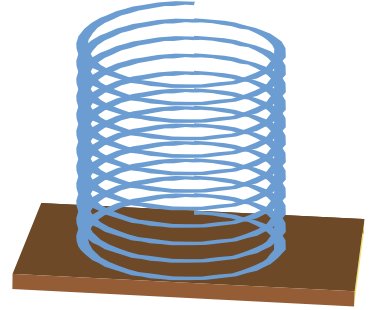
## Lab 4 (associated with Hw 4): Period of vibration of a slinky

This laboratory develops physical intuition for 2<sup>nd</sup>-order, linear, ODEs and associated quantities, including natural length, equilibrium length, period of vibration, natural frequency, decay ratio, and damping ratio.

### Lab 4.1 Period of vibration of a massless slinky

To analytically estimate the period of vibration of a mass-spring system (“massless” slinky with attached massive object), gather the following data:

Quantity	Symbol	Value
Mass of object attached to slinky	$m_{object}$	_____ kg
Stretch when object is attached	$stretch$	_____ m
Earth’s gravitational acceleration	$g$	$9.8 \frac{m}{s^2}$



Assuming that  $\zeta \approx 0$  (little damping) the period of vibration  $\tau_{period}$  can be estimated as follows:

$$k = \frac{m_{object} g}{stretch} = \text{_____} \frac{N}{m} \qquad \omega_n \stackrel{(5.3)}{=} \sqrt{\frac{k}{m_{object}}} = \text{_____} \frac{rad}{sec}$$

$$\tau_{period_{analytical}} \stackrel{(1)}{=} \frac{2\pi}{\omega_d} \stackrel{(5.5)}{\approx} \frac{2\pi}{\omega_n} = \text{_____} sec$$

To experimentally determine  $\tau_{period}$ , measure several periods of vibration and then average them, i.e.,

$$\tau_{period_{experimental}} = \frac{\text{_____} sec}{\text{_____} cycles} = \text{_____} \frac{sec}{cycle}$$

The error in the analytical estimate of  $\tau_{period}$  to the experimental one is

$$\%Error = 100 * \frac{\tau_{period_{analytical}} - \tau_{period_{experimental}}}{\tau_{period_{experimental}}} = 100 * \frac{\text{_____} - \text{_____}}{\text{_____}} = \text{_____} \%$$

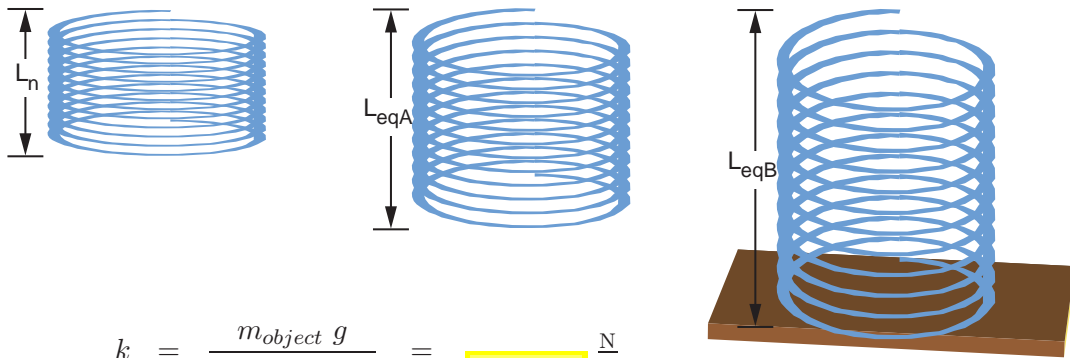
### Lab 4.2 A better model: Period of vibration of a massive slinky

The previous experiment showed how to experimentally determine the period of vibration ( $\tau_{\text{period}}$ ) of a massive object suspended by a “massless” slinky. To account for modeling errors associated with the mass of the slinky, the period of vibration is recalculated with the following data:

Quantity	Symbol	Value
Mass of object attached to slinky	$m_{\text{object}}$	_____ kg
<b>Natural length</b> of slinky ( <b>completely</b> unstretched)	$L_n$	_____ m
<b>Equilibrium length</b> of vertical slinky <b>without</b> attached object	$L_{\text{eqA}}$	_____ m
<b>Equilibrium length</b> of vertical slinky <b>with</b> attached object	$L_{\text{eqB}}$	_____ m
Earth’s gravitational acceleration	$g$	$9.8 \frac{\text{m}}{\text{s}^2}$

By using the equations which govern the static equilibrium lengths of the slinky (with and without the attached object), one can estimate  $\tau_{\text{period}}$ . The process consists of the following steps.

- Measure  $L_n$ , the slinky’s **natural length** (the slinky’s length when it is **completely** unstretched).
- This experiment models the slinky’s mass as a particle of mass  $m_{\text{slinky}}$  attached to the end of the slinky. This is the mass required to stretch the slinky to  $L_{\text{eqA}}$ . Carefully measure  $L_{\text{eqA}}$ .
- Attach an object of known mass to the slinky. Measure  $L_{\text{eqB}}$  (the new **equilibrium length**).
- **Show** how to use  $\vec{F} = m \vec{a}$  to calculate  $k$  (the slinky’s linear spring constant) and  $m_{\text{slinky}}$ .



$$k = \frac{m_{\text{object}} g}{L_{\text{eqB}} - L_{\text{eqA}}} = \text{_____} \frac{\text{N}}{\text{m}}$$

$$m_{\text{slinky}} = \frac{k}{g} (L_{\text{eqA}} - L_n) = \text{_____} \text{ kg}$$

- Calculate the motion’s **natural frequency**  $\omega_n$  and  $\tau_{\text{period}_{\text{analytical}}}$  (the mathematically determined **period of vibration**).

$$\omega_n = \sqrt{\frac{k}{m_{\text{slinky}} + m_{\text{object}}}} = \text{_____} \frac{\text{rad}}{\text{sec}}$$

$$\omega_d \stackrel{\triangle}{(5.5)} \omega_n \sqrt{1 - \zeta^2} \approx \omega_n \quad (\text{when } \zeta \text{ is small})$$

$$\tau_{\text{period}_{\text{analytical}}} \stackrel{(6.1)}{=} \frac{2\pi}{\omega_d} \approx \text{_____} \text{ sec}$$

- Measure  $\tau_{\text{period}_{\text{experimental}}}$  (the experimentally determined **period of vibration**) by timing several periods of vibration and then averaging them.

$$\tau_{\text{period}_{\text{experimental}}} = \frac{\text{_____} \text{ sec}}{\text{_____} \text{ cycles}} = \text{_____} \frac{\text{sec}}{\text{cycle}}$$

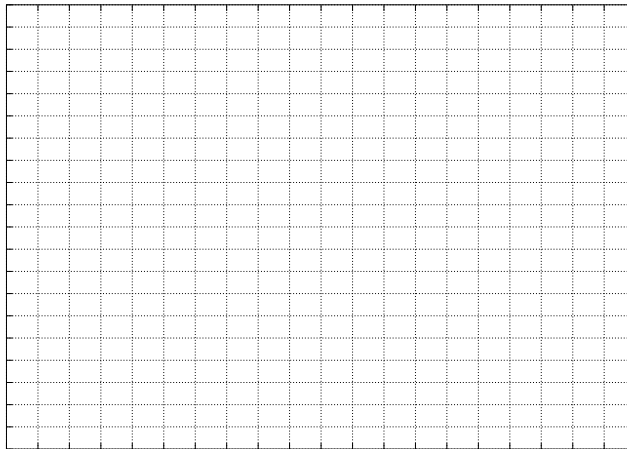
- Calculate the percentage error in the analytical determination of  $\tau_{\text{period}}$ .

$$\text{Error} = 100 \frac{\tau_{\text{period}_{\text{experimental}}} - \tau_{\text{period}_{\text{analytical}}}}{\tau_{\text{period}_{\text{experimental}}}} = \underline{\hspace{2cm}} \%$$

- Using the same data, calculate the percentage error in the analytical determination of  $\tau_{\text{period}}$  when  $m_{\text{spring}}$  is assumed to be zero.

$$\text{Error} = 100 \frac{\tau_{\text{period}_{\text{experimental}}} - \tau_{\text{period}_{\text{analytical}}}}{\tau_{\text{period}_{\text{experimental}}}} = \underline{\hspace{2cm}} \%$$

- The error in calculating  $\tau_{\text{period}}$  is **smaller/larger** (circle one) when one assumes  $m_{\text{spring}} = 0$ .
- Make a rough sketch that is useful for experimentally measuring the *decay ratio*.



$$\text{decayRatio} \approx \underline{\hspace{2cm}}$$

- Make a rough estimate of the *damping ratio*  $\zeta$ . (See Homework 4.6 for details).

$$\zeta \approx \underline{\hspace{2cm}}$$