12.1 Matrix rows and columns

Given: $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Row 1 of M $= \begin{bmatrix} 1 & 2 \end{bmatrix}$

Row 2 of M $= \begin{bmatrix} 3 & 4 \end{bmatrix}$

$M_{2,1} = 3$

Column 1 of M $= \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Column 2 of M $= \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

12.2 Matrix transpose

$$\text{Transpose} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$\text{Transpose} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$$

12.3 Matrix addition and subtraction (+ , -)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} - \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} = \begin{bmatrix} a-2 & b-4 & c-6 \\ d-3 & e-5 & f-7 \end{bmatrix}$$

12.4 Scalar-matrix multiplication (∗)

$$5 * \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$$

$$5 * \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 5a & 5b & 5c \\ 5d & 5e & 5f \end{bmatrix}$$

12.5 Matrix-matrix multiplication (∗)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 3a+5b \\ 3c+5d \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 3 & x \\ 5 & y \end{bmatrix} = \begin{bmatrix} 3a+5b & ax+by \\ 3c+5d & cx+dy \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} * \begin{bmatrix} x & y \end{bmatrix} = \begin{bmatrix} ax & ay \\ bx & by \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} * \begin{bmatrix} x & 3 \\ y & 5 \\ z & 7 \end{bmatrix} = \begin{bmatrix} ax+by+cz & 3a+5b+7c \\ dx+ey+fz & 3d+5e+7f \end{bmatrix}$$

12.6 Matrix determinants

$\det \begin{bmatrix} 5 \end{bmatrix} \triangleq 5$

$\det \begin{bmatrix} a \end{bmatrix} \triangleq a$

$$\det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = 1*4 - 2*3 = \text{-}2$$

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ab - cd$$

Calculate the following determinant three ways, namely by expanding along the first row, expanding along the first column, and expanding along the second row.

$$\det \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 9 \end{bmatrix} = {}^{+}1 \det \begin{bmatrix} 5 & 6 \\ 0 & 9 \end{bmatrix} + \text{-}2 \det \begin{bmatrix} 4 & 6 \\ 7 & 9 \end{bmatrix} + {}^{+}3 \det \begin{bmatrix} 4 & 5 \\ 7 & 0 \end{bmatrix} = \text{-}48$$

$$= {}^{+}1 \det \begin{bmatrix} 5 & 6 \\ 0 & 9 \end{bmatrix} + \text{-}4 \det \begin{bmatrix} 2 & 3 \\ 0 & 9 \end{bmatrix} + {}^{+}7 \det \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix} = \text{-}48$$

$$= \text{-}4 \det \begin{bmatrix} 2 & 3 \\ 0 & 9 \end{bmatrix} + {}^{+}5 \det \begin{bmatrix} 1 & 3 \\ 7 & 9 \end{bmatrix} + \text{-}6 \det \begin{bmatrix} 1 & 2 \\ 7 & 0 \end{bmatrix} = \text{-}48$$

Calculate the following determinant by expanding along the third column.

$$\det \begin{bmatrix} a & b & c \\ d & e & 0 \\ g & h & 0 \end{bmatrix} = {}^{+}c * \det \begin{bmatrix} d & e \\ g & h \end{bmatrix} = c(dh - eg)$$

12.7 Optional: Matrix inverse with determinants and adjugate (adjunct) matrices

Calculate the following matrix inverses (the 3×3 matrix inverses are optional).

$$\text{inv}\begin{bmatrix} 5 \end{bmatrix} = \frac{\text{adj}\begin{bmatrix} 5 \end{bmatrix}}{\det\begin{bmatrix} 5 \end{bmatrix}} = \frac{\begin{bmatrix} 1 \end{bmatrix}}{5} = \begin{bmatrix} 0.2 \end{bmatrix} \qquad \text{inv}\begin{bmatrix} a \end{bmatrix} = \begin{bmatrix} \dfrac{1}{a} \end{bmatrix}$$

$$\text{inv}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \frac{\text{adj}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}{\det\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}} = \frac{\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}}{-2} = \begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

$$\text{inv}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \frac{\text{adj}\begin{bmatrix} a & b \\ c & d \end{bmatrix}}{\det\begin{bmatrix} a & b \\ c & d \end{bmatrix}} = \frac{\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}{a\,d - b\,c}$$

$$\text{inv}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 9 \end{bmatrix} = \begin{bmatrix} -0.9375 & 0.375 & 0.0625 \\ -0.125 & 0.25 & -0.125 \\ 0.7291667 & -0.2916667 & 0.0625 \end{bmatrix}$$

$$\text{inv}\begin{bmatrix} a & b & c \\ d & e & 0 \\ g & h & 0 \end{bmatrix} = \begin{bmatrix} 0 & h/(d*h - e*g) & -e/(d*h - e*g) \\ 0 & -g/(d*h - e*g) & d/(d*h - e*g) \\ 1/c & -(a*h - b*g)/(c*(d*h - e*g)) & (a*e - b*d)/(c*(d*h - e*g)) \end{bmatrix}$$

12.8 Matrix form of scalar equations (matrix multiplication in reverse)

Put the following sets of scalar equations into matrix form.

$a\,x + b\,y = 12$	$a\,x + b\,y + c\,z = 12$	$a\,x + b\,y - 12 = 0$
$d\,x + e\,y = 15$	$d\,x + e\,y + f\,z = 15$	$d\,x + e\,y - 15 = 0$
$\begin{bmatrix} a & b \\ d & e \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$	$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$	$\begin{bmatrix} x & y & 0 & 0 \\ 0 & 0 & x & y \end{bmatrix}\begin{bmatrix} a \\ b \\ d \\ e \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \end{bmatrix}$

12.9 Optional: Solving sets of linear algebraic equations.

$$a\,x + b\,y = 1$$
$$d\,x + e\,y = 2$$

$$a\,x + b\,y + c\,z = 1$$
$$2\,x + 3\,y + 4\,z = 2$$
$$2\,x + 4\,y + 6\,z = 4$$

Solve for x, y:

$$x = \frac{e - 2\,b}{a\,e - b\,d} \qquad y = \frac{2\,a - d}{a\,e - b\,d}$$

Solve for x, y, z:

$$x = \frac{-1 - 2\,b + 2\,c}{2\,b - a - c} \qquad y = \frac{2 + 2\,a - 2\,c}{2\,b - a - c} \qquad z = \frac{-1 - 2\,a + 2\,b}{2\,b - a - c}$$

12.10 Optional: Matrix computation with MotionGenesis and/or MATLAB®

Use **M**otion**G**enesis and/or MATLAB® to do Homework 12.1 - Homework 12.9 (include optional problems but exclude Homework 12.8). Print and submit a **M**otion**G**enesis input command file named `MatrixAlgebra.txt` that uses the following commands.

GetElement	GetRow	GetColumn	GetTranspose
+ − *	GetDeterminant	GetInverse	Solve