12.1 Matrix rows and columns

Given: $M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ Row 1 of M $= [\ \square \ \ \square\ ]$ Row 2 of M $= [\ \square \ \ \square\ ]$

$M_{2,1} = \square$ Column 1 of M $= \begin{bmatrix} \square \\ \square \end{bmatrix}$ Column 2 of M $= \begin{bmatrix} \square \\ \square \end{bmatrix}$

12.2 Matrix transpose

$$\text{Transpose} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \qquad \text{Transpose} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \square$$

12.3 Matrix addition and subtraction (+ , -)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \qquad \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} - \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} = \square$$

12.4 Scalar-matrix multiplication ($*$)

$$5 * \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} \qquad 5 * \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \square$$

12.5 Matrix-matrix multiplication ($*$)

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} \square \\ \square \end{bmatrix} \qquad \begin{bmatrix} a & b \\ c & d \end{bmatrix} * \begin{bmatrix} 3 & x \\ 5 & y \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} * \begin{bmatrix} x & y \end{bmatrix} = \square \qquad \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} * \begin{bmatrix} x & 3 \\ y & 5 \\ z & 7 \end{bmatrix} = \square$$

12.6 Matrix determinants

$\det \begin{bmatrix} 5 \end{bmatrix} \triangleq 5$

$\det \begin{bmatrix} a \end{bmatrix} \triangleq \square$

$$\det \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \square * \square - \square * \square = -2 \qquad \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \square$$

Calculate the following determinant three ways, namely by expanding along the first row, expanding along the first column, and expanding along the second row.

$$\det \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 0 & 9 \end{bmatrix} = +1 \det \begin{bmatrix} 5 & 6 \\ 0 & 9 \end{bmatrix} + -2 \det \begin{bmatrix} 4 & 6 \\ 7 & 9 \end{bmatrix} + +3 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \square$$

$$= +1 \det \begin{bmatrix} 5 & 6 \\ 0 & 9 \end{bmatrix} + -4 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + +7 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \square$$

$$= -4 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + +5 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} + -6 \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \square$$

Calculate the following determinant by expanding along the third column.

$$\det \begin{bmatrix} a & b & c \\ d & e & 0 \\ g & h & 0 \end{bmatrix} = \square * \det \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix} = \square$$

12.7 Optional: Matrix inverse with determinants and adjugate (adjunct) matrices

Calculate the following matrix inverses (the 3×3 matrix inverses are optional).

$$\text{inv}\begin{bmatrix}5\end{bmatrix} = \frac{\text{adj}\begin{bmatrix}5\end{bmatrix}}{\det\begin{bmatrix}5\end{bmatrix}} = \frac{\begin{bmatrix}1\end{bmatrix}}{5} = \begin{bmatrix}0.2\end{bmatrix} \qquad \text{inv}\begin{bmatrix}a\end{bmatrix} = \boxed{}$$

$$\text{inv}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} = \frac{\text{adj}\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}}{\det\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}} = \frac{\begin{bmatrix}4 & -2\\ -3 & 1\end{bmatrix}}{\boxed{}} = \boxed{}$$

$$\text{inv}\begin{bmatrix}a & b\\ c & d\end{bmatrix} = \frac{\text{adj}\begin{bmatrix}a & b\\ c & d\end{bmatrix}}{\det\begin{bmatrix}a & b\\ c & d\end{bmatrix}} = \frac{\boxed{}}{\boxed{}}$$

$$\text{inv}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 0 & 9\end{bmatrix} = \begin{bmatrix}-0.9375 & 0.375 & 0.0625\\ -0.125 & 0.25 & -0.125\\ 0.7291667 & -0.2916667 & 0.0625\end{bmatrix}$$

$$\text{inv}\begin{bmatrix}a & b & c\\ d & e & 0\\ g & h & 0\end{bmatrix} = \begin{bmatrix}0 & h/(d*h-e*g) & -e/(d*h-e*g)\\ 0 & -g/(d*h-e*g) & d/(d*h-e*g)\\ 1/c & -(a*h-b*g)/(c*(d*h-e*g)) & (a*e-b*d)/(c*(d*h-e*g))\end{bmatrix}$$

12.8 Matrix form of scalar equations (matrix multiplication in reverse)

Put the following sets of scalar equations into matrix form.

$a\,x + b\,y = 12$ $d\,x + e\,y = 15$ $\boxed{}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}12\\ 15\end{bmatrix}$	$a\,x + b\,y + c\,z = 12$ $d\,x + e\,y + f\,z = 15$ $\boxed{}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}12\\ 15\end{bmatrix}$	$a\,x + b\,y - 12 = 0$ $d\,x + e\,y - 15 = 0$ $\boxed{}\begin{bmatrix}a\\ b\\ d\\ e\end{bmatrix} = \begin{bmatrix}\boxed{}\\ 15\end{bmatrix}$

12.9 Optional: Solving sets of linear algebraic equations.

$$a\,x + b\,y = 1$$
$$d\,x + e\,y = 2$$

$$a\,x + b\,y + c\,z = 1$$
$$2\,x + 3\,y + 4\,z = 2$$
$$2\,x + 4\,y + 6\,z = 4$$

Solve for x, y:

$$x = \frac{e - 2\,b}{a\,e - b\,d} \qquad y = \frac{2\,a - d}{a\,e - b\,d}$$

Solve for x, y, z:

$$x = \frac{-1 - 2\,b + 2\,c}{2\,b - a - c} \qquad y = \frac{2 + 2\,a - 2\,c}{2\,b - a - c} \qquad z = \frac{-1 - 2\,a + 2\,b}{2\,b - a - c}$$

12.10 Optional: Matrix computation with MotionGenesis and/or MATLAB®

Use **M**otion**G**enesis and/or MATLAB® to do Homework 12.1 - Homework 12.9 (include optional problems but exclude Homework 12.8). Print and submit a **M**otion**G**enesis input command file named `MatrixAlgebra.txt` that uses the following commands.

GetElement	GetRow	GetColumn	GetTranspose
+ − *	GetDeterminant	GetInverse	Solve