Chapter 14

Complex numbers

$$i \triangleq \sqrt{-1}$$

$$e^{i\,\theta} = \cos(\theta) + i\,\sin(\theta)$$

Motivating questions (see examples in Hw 8)

- The following proof involves the imaginary number i defined as $i \triangleq \sqrt{\text{-}1}$ and shows the surprising conclusion that $1 = \text{-}1$. Circle the incorrect step in the proof and explain your reasoning.

$$1 \;=\; \sqrt{1} \;=\; \sqrt{(\text{-}1)^2} \;=\; \sqrt{\text{-}1} * \sqrt{\text{-}1} \;=\; i * i \;=\; i^2 \;=\; \text{-}1$$

- Find all real and/or complex numbers that can appear on the right-hand side of the equal signs.[1]

1^4	=		=		=		=	1
$1^{1/4}$	=		=		=		=	$\pm 1,\ \pm i$
$1^{1/3}$	=		=		=		=	$1,\ \text{-}0.5 \pm 0.866\, i$

Summary

Complex numbers are useful in many aspects of dynamic systems, including:

- Solving 2^{nd}-order, 3^{rd}-order, and higher-order ODEs
- Root locus
- Control system design
- Frequency response
- Eigen-analysis

This chapter provides basic mathematical tools for complex numbers, namely addition, multiplication, division, and exponentiation.[2]

[1] Answers at www.MotionGenesis.com ⇒ Textbooks ⇒ Resources. Alternate: $1^{1/4} = \sqrt{\sqrt{1}} = \sqrt{\pm 1} = \pm 1$ or $\pm i$.

[2] Complex numbers are said to be "***closed***" under addition, subtraction, negation, multiplication, division, and exponentiation because when these operations are performed on complex numbers, only complex numbers result. Complex numbers are said to be "***algebraically closed***" because polynomial equations with complex number coefficients can only produce complex numbers. Real numbers are not closed under exponentiation. For example, $\text{-}4^{0.5} = \sqrt{\text{-}4} = \pm 2\, i$ produces a complex (not real) number. Real numbers are not algebraically closed. For example, although the polynomial equation $x^2 + 2\,x + 5 = 0$. has real coefficients, its roots are the complex numbers $x = \text{-}1 \pm 2\, i$.